EASTERN UNIVERSITY, SRI LANKA
FACULTY OF SCIENCE
First Year Second Semester Examination in Science - 2021/2022
(Aug./Sept., 2024)
MT 1232 - Mathematical Methods

Answer All Questions   Time Allowed : Two Hours

Q1. (a) Define what is meant by the point $x = x_0$, being

     i. *an ordinary;*

     ii. *a singular;*

     iii. *a regular singular*

     point of the ordinary differential equation (ODE)

$$y'' + p(x)y' + q(x)y = 0,$$

     where prime denotes differentiation with respect to x, and $p(x)$ and $q(x)$ are rational functions.

[30 Marks]

(b)   i. Find the singular points of the ODE $(x^2 - 4)^2 y'' + 3(x - 2)y' + 5y = 0$ and find whether the points are regular or irregular. [30 Marks]

     ii. Find the general solution of

$$4xy'' + 2y' - 7y = 0$$

     using the method of Frobenius. [40 Marks]

Q2. (a)   i. Define the *gamma-function* $\Gamma(x)$ and the *beta-function* $B(m, n)$, where m, n are positive integers. [20 Marks]

     ii. Evaluate the integral $\int_0^4 \dfrac{x^3}{\sqrt{4 - x}} dx$.

     (You may use the following results without proof $B(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m + n)}$.)

[30 Marks]

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(b)   i. Eliminate the arbitrary constants a and b from

$$z^2 \cot^2(\alpha) = (x - a)^2 + (y - b)^2,$$

where α is an arbitrary constant.   [25 Marks]

ii. Eliminate the arbitrary functions $f(x)$ and $g(y)$ from $z = yf(x) + xg(y)$.

[25 Marks]

Q3.   (a) Find the general solution of the partial differential equation (PDE) using the Lagrange's method

$$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} + x^2 - y^2 = 0.$$

[50 Marks]

(b) Find the solution of the PDE using the separation of variables,

$$2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 4u.$$

[50 Marks]

Q4.   (a) Define the *Laplace* and *Inverse Laplace* transformations.   [10 Marks]

(b) Find the following, where $\mathcal{L}$ is the Laplace transformation:

i. $\mathcal{L}[a]$, where a is a constant;

ii. $\mathcal{L}[e^{at}]$;

iii. $\mathcal{L}^{-1}\left(\dfrac{2s - 1}{s^3 - s}\right)$.

[30 Marks]

(c)   i. Solve the initial value problem using the Laplace transformation;

$$x''(t) - 3x'(t) + 2x(t) = 4e^{2t},$$

subject to the initial conditions $x(0) = -3$, $x'(0) = 5$.   [30 Marks]

ii. Solve the system of the simultaneous ODEs by using the Laplace transformation:

$$x' = x - 2y,$$
$$y' = 5x - y$$

subject to the initial conditions; $x(0) = -1$, $y(0) = 2$.   [30 Marks]

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