EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST YEAR EXAMINATION IN SCIENCE - 2015/2016

SECOND SEMESTER - (MAY/JUNE, 2018)

AM 104 - DIFFERENTIAL EQUATIONS

AND

FOURIER SERIES

Answer **All** Questions | Time Allowed: **3** Hours

Q1. (a) State the necessary and sufficient condition for the ordinary differential equation (ODE)

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

to be *exact*. [**10 Marks**]

Find the general solution of the ODE

$$xy^2 \sinh x + y^2 \cosh x + (e^y + 2xy\cosh x)\frac{dy}{dx} = 0.$$

[**50 Marks**]

(b) Solve the nonlinear first-order Bernoulli's equation

$$\frac{dy}{dx} + \sqrt{x}y - \frac{2}{3}\sqrt{\frac{x}{y}} = 0.$$

[**40 Marks**]

Q2. Let $D \equiv d/dx$ be a differential operator. Obtain the general solution of the following ODEs:

(i) $(D^2 + 2D + 4)y = e^x \sin 2x$;

(ii) $(D^3 - 5D^2 - D + 5)y = 10x - 63e^{-2x} + 29 \sin 2x$.

[100 Marks]

Q3. (i) Find the general solution of the Cauchy-Euler differential equation

$$(x^2D^2 - 2xD + 2)y = x^2 + 1 \quad \text{for } x > 0.$$

[50 Marks]

(a) Define what is meant by *orthogonal trajectories* of curves.

[10 Marks]

Find the orthogonal trajectories of the family of curves

$$r = a(1 + \sin\theta)$$

in polar coordinates, where a is a constant. [40 Marks]

Q4. (a) Define what is meant by the point, $x = x_0$, being

(i) an *ordinary* ;

(ii) a *singular*;

(iii) a *regular singular*

point of the DE

$$y'' + p(x)y' + q(x)y = 0,$$

where the prime denotes differentiation with respect to x, and $p(x)$ and $q(x)$ are rational functions.

[30 Marks]

(b) (i) Find the regular singular point(s) of the ODE

$$4xy'' + 2y' + y = 0. \qquad (1)$$

(ii) Use the method of Frobenius to find the general solution of the equation (1).

[70 Marks]

Q5. (a) Solve the following system of DEs:

(i) $\dfrac{x^2dx}{y^3} = \dfrac{y^2dy}{x^3} = \dfrac{dz}{z}$;

(ii) $\dfrac{dx}{x^2 + y^2 - yz} = \dfrac{dy}{-x^2 - y^2 + xz} = \dfrac{dz}{(x - y)z}$.

[30 Marks]

(b) Write down the condition of integrability of the total differential equation

$$P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz = 0.$$

[5 Marks]

Hence solve the following equation

$$(y^2+z^2+2xy+2xz)dx+(x^2+z^2+2xy+2yz)dy+(x^2+y^2+2xz+2yz)dz = 0.$$

[15 Marks]

(c) Find the equation of the integral surface satisfying the linear partial differential equation (PDE)

$$4yz\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + 2y = 0,$$

and passing through the curve, $y^2 + z^2 = 1,\ x + z = 2$.

[30 Marks]

(d) Apply Charpit's method or otherwise to find the complete and the singular solution of the nonlinear first-order PDE

$$p(1 - q^2) - q(1 - z) = 0.$$

Here, $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$.

[20 Marks]

Q6. Expand, $f(x) = x^2,\ 0 < x < 2\pi$, in a Fourier series if

(a) the period is 2π;

(b) the period is not specified.

[100 Marks]

* * * * * * * * * **