



EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
FIRST YEAR EXAMINATION IN SCIENCE - 2016/2017
FIRST SEMESTER (Aug. /Sept., 2018)
AM 151 - MATHEMATICA

Answer all questions

Time allowed: Two hours

Q1. (a) Enter the appropriate Mathematica commands for the following:

- list down the first 20 triangular numbers;
- list down the multiples of eight between the range of 1 to 100;
- combine those two sets obtained in above parts (part i. and part ii.);
- discover all odd numbers from the combined list (part iv).

(b) i. Factorize the expression $4x^{\frac{2}{3}} + 8x^{\frac{1}{3}} + 4$.

ii. Simplify the given expression $\frac{(\frac{2}{x} - 3)}{(1 - \frac{1}{x-1})}$.

(c) i. Evaluate $\int \frac{x^5 + x^2 + x + 2}{(x^2 + 1)^2} dx$.

ii. Evaluate $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$.

iii. Find the third derivative of the function $g(t) = t^3 - \sqrt{t} + e^{-2t}$, with respect to t .

(d) Consider the equation, $x^2 + yx + 3 = 0$.

i. Find the solution of this given equation.

ii. Use one of the above solutions to extract it from the list of solutions and replace the value of x as 1 to obtain the output.

Q2.(a) Find the partial derivatives of $(x^3 + y^2)$ with respect to x and y .

(b) Find the absolute maximum and minimum values of the function,
 $f(x) = x^4 - 4x^3 + 2x^2 + 4x + 2$ on the interval $[0, 4]$.

(c) Consider the sequence $\left\{ \frac{n+2}{2^n+1} (-2)^{n-1} \right\}$, where $n = 1$ to ∞ .

i. List the first ten terms of the sequence.

ii. Find the sum of the first ten terms of the sequence.

iii. Find the sum of the first n terms of the sequence.

iv. Determine whether this series converges or diverges.

(d) Let $M = \begin{pmatrix} \frac{1}{10} & \frac{2}{10} & \frac{7}{10} \\ \frac{3}{10} & \frac{2}{10} & \frac{4}{10} \\ \frac{5}{10} & \frac{4}{10} & \frac{1}{10} \end{pmatrix}$.

Compute $\lim_{n \rightarrow \infty} M^n$ and display the answer in a matrix format.

Q3.(a) Sketch the graph of $f(x) = x^4 - 50x^2 + 300$ and its derivative, on one set of axes for $-10 \leq x \leq 10$, $-500 \leq y \leq 500$.

(b) A wire, 100 inches long, is to be used to form a square and a circle. Determine how the wire should be distributed in order for the combined area of the two figures to be, as large as possible and as small as possible.

(c) Plot the graph of

$$g(x) = \begin{cases} x, & x < 1 & \text{for } 0 \leq x \leq 2; \\ x^3, & x \geq 1 & \text{for } 0 \leq x \leq 2. \end{cases}$$

(d) The equation $r = \sin n\theta$, where n is a positive integer, represents a family of polar curves called roses. Investigate the behavior of this family and form a conjecture about how the number of loops is related to n .

(e) Graph the intersection of the paraboloid $z = x^2 + y^2$ with the plane $y + z = 2$. Obtain a front view and a side view.