Answer all questions   Time: One hour

1. (a) Simplify the following:

   i. $\dfrac{(-a^4b)^3(ab)^5}{-a^8b^8}$;

   ii. $\left(\dfrac{2a^7b^{-4}}{8a^9b^{-2}}\right)^{-3} \times (-6a^{-1}b^0)^{-2}$.

   (b) Solve the following equations:

   i. $3^{2x} - 4 \times 3^{x+1} + 3^3 = 0$;

   ii. $y = 2\log 30 + 4\log 2 - 2\log 12$.

   (c) If $x^2 + 9y^2 = 15xy$ then prove that $2\log\left(\dfrac{x-3y}{3}\right) = \log x + \log y$.

   (d) Find the equation of a straight line that passing through $\left(\dfrac{5}{3}, 2\right)$ and parallel to the straight line $y = \dfrac{7}{2}x - 3$.

   (50 Marks)

2. (a) Evaluate the following limits:

   i. $\lim\limits_{x \to \infty} \left(\dfrac{1 - 5x^2 + 7x^3 - 49x^5}{2 + x + 7x^5} \right)$;

   ii. $\lim\limits_{x \to 0} \left(\dfrac{x^2 - 3}{x - \sqrt{3}} \right)$;

   iii. $\lim\limits_{x \to 0} \left(\dfrac{\sqrt{1 + x} - 1}{x} \right)$.

(b) Differentiate the following with respect to x:

   i. $y = x^5 - 3x^2 = 7$;

   ii. $y = e^{2x} \sin 3x$;

   iii. $y = \dfrac{x^2 - 2}{x - 1}$.

(c) Integrate the following:

   i. $\displaystyle\int \dfrac{1}{x \ln x}\, dx$;

   ii. $\displaystyle\int \dfrac{x}{\sqrt{3x^2 - 1}}\, dx$.

(d) Find the maximum and minimum points of a function $y = x^3 - 6x^2 + 9x - 2$.

(50 Ma