EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE, 2010/2011

FIRST SEMESTER (Nov./Dec., 2012)

AM 106 - TENSOR CALCULUS

(Proper & Repeat)

Answer all questions **Time : One hour**

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1. (a) Explain what is meant by the following terms:

   i. Covariant tensor;

   ii. Contravariant tensor.

   (b) Write down the law of transformation for the following tensors:

   i. A_{mn};

   ii. B_{r}^{pq};

   iii. C_{rt}^{pqs}.

   (c) If $ds^2 = g_{ij}dx^i dx^j$ is an invariant, show that g_{ij} is a symmetric covariant tensor of rank two.

   (d) Express the relationship between the following associated tensors:

   i. A^{jkl} and A_{pqr};

   ii. $A_{j\ \cdot\ l}^{\ k}$ and A^{qkr}.

(e) If $X(i,j)\, B^j = C_i$, where B^j is an arbitrary contravariant vector and C_i is a covariant vector, then show that $X(i,j)$ is a tensor. What is its rank and type.

2. (a) Define the following:

   i. Christoffel's symbols of the first and second kind;

   ii. Geodesic;

   iii. Covariant derivative of A_p.

   (b) With the usual notations, prove the following:

   i. $[pq, r] = g_{rs}\Gamma^s_{pq}$;

   ii. $[pm, q] + [qm, p] = \dfrac{\partial g_{pq}}{\partial x^m}$;

   iii. $\dfrac{\partial g^{pq}}{\partial x^m} + g^{pn}\Gamma^q_{mn} + g^{qn}\Gamma^p_{mn} = 0$.

   Hence show that,

$$g_{jk;q} = 0.$$

   (c) Show that the non-vanishing Christoffel's symbols of the second kind in cylindrical coordinate (ρ, ϕ, z) are given by

$$\Gamma^1_{22} = -\rho, \quad \Gamma^2_{21} = \frac{1}{\rho}, \quad \Gamma^2_{12} = \frac{1}{\rho},$$

   where $x^1 = \rho, \ x^2 = \phi, \ x^3 = z$.