



EASTERN UNIVERSITY, SRI LANKA
FIRST YEAR EXAMINATION IN SCIENCE - 2010/2011
FIRST SEMESTER (Nov/Dec., 2012)
AM 151 - MATHEMATICA
(PROPER & REPEAT)

Answer all questions

Time: Two hours

- Q1. (a) Find the greatest integer less than or equal to π^2 . [10 marks]
(b) Check whether the number 56841 is prime or not? [5 marks]
(c) Compute a numerical value for the sum

$$\left(1 + \frac{1}{2} + \dots + \frac{1}{5}\right) + \left(2 + 1 + \dots + \frac{2}{5}\right) + \dots + (10 + 5 + \dots + 2).$$

[20 marks]

- (d) Find the value of $\cosh^2 x - \sinh^2 y$ by replacing suitable random numbers for x and y . [20 marks]
(e) Factorize the expression $x^4 + 6x^3 + 9x^2 - 4x - 12$. [10 marks]
(f) Solve the system of equations:

$$2x + 3y + z = 1,$$

$$3x + y - 3z = 2,$$

$$x - y + 6z = 0.$$

[20 marks]

- (g) Define the Heaviside function:

$$H(x) = \begin{cases} 1, & x > 0, \\ 0, & x \leq 0, \end{cases}$$

using **Which** statement, and evaluate at $x = 5$.

[15 marks]

Q2. (a) Consider the following table assigned with titles n , \sqrt{n} and $\sqrt[3]{n}$ of the first, second and third column, respectively. Moreover, the first column represents the possible values of n , and the corresponding values of \sqrt{n} and $\sqrt[3]{n}$ are given in the second and third columns, respectively. Also, the numbers created in the table are all centered.

n	\sqrt{n}	$\sqrt[3]{n}$
1	1.	1.
2	1.41421	1.25992
3	1.73205	1.44225
4	2.	1.5874
5	2.23607	1.70998

Create a suitable list without typing manually the numbers given in the table, and apply the appropriate commands to display the table as exactly the same as above. Furthermore, modify the above table so that the all numbers are to be rounded to 4-decimal points and to be allocated for 5-digits future spaces to the left of each number. [30 marks]

(b) i. Sketch the graph defined by an equation:

$$y^2 = x^3(2 - x), \quad 0 \leq x \leq 2,$$

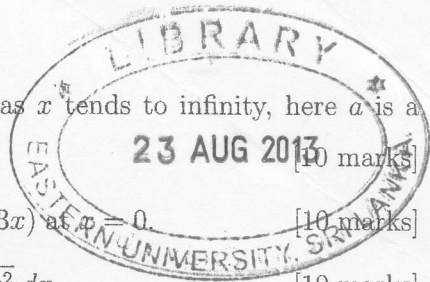
with thickness scaled at 0.02.

ii. Using the **Table** command create a list of 50 random coordinates, (x, y, z) , consist of integers in the interval $(1,10)$, and plot them in 3-dimension. [45 marks]

(c) Obtain the surface of revolution by rotating the curve,

$$z = x^2, \quad 0 \leq x \leq 1,$$

about the z -axis. Further, assign the axes with names x, y and z . Moreover, find the surface of revolution about x -axis, when rotating the curve given above. [25 marks]



- Q3. (a) Find the limit of the function: $1 + \left(\frac{a}{x}\right)^x$, as x tends to infinity, here a is a constant. [10 marks]
- (b) Determine the third derivative of $\cos(x) \sin(3x)$ at $x = 0$. [10 marks]
- (c) Solve for an approximate value of $\int_0^1 \sqrt{\cos x^2} dx$. [10 marks]
- (d) Solve the system of differential equations:

$$\frac{dx}{dt} = 2x - 5y,$$

$$\frac{dy}{dt} = x + 4y,$$

subject to the initial conditions: $x(0) = 2$ and $y(0) = -1$. [20 marks]

- (e) i. The recursive relation: $c_n = c_{n-1} + c_{n-2}$, $n \geq 2$, where $c_0 = 1$ and $c_1 = 1$, can be used to find fibonacci numbers. Identify the errors in the following coding, and correct them to find 20th fibonacci number.

```
fib[n] = Module[s,
```

```
If[n = 0 && n = 1, s = 1, s = fib(n - 1) + fib(n - 2)]
```

```
return[s]
```

```
]
```

[20 marks]

- ii. Write a simple programme to find the sum of numbers using **Module** command and **For** loop. Create a list of odd numbers from 5 to 25, and check your coding using the list you created. [30 marks]