Answer all questions   Time: One hour

1. (a) Simplify the following:

   i. $\left(\dfrac{8abc^2}{27a^2b^3c}\right) \div \left(\dfrac{4a^2b}{9c^3}\right) \times \left(\dfrac{bc^{-1}}{a^{-2}}\right)^2$,

   ii. $\dfrac{\log_4\left(\dfrac{k}{2}\right) + \log_4 2}{\log_{64} k}$.

   (b) Factorize the following:

   i. $9x^2 - 12x + 4$,

   ii. $ab(x^2 - y^2) - xy(a^2 - b^2)$.
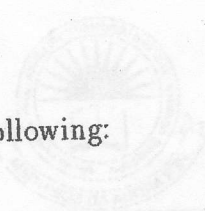
   (c) Solve the following equations.

   i. $3 \times 9^{2x-1} = 27^{-x}$,

   ii. $\log(5x - 6) + \log(2x + 3) = \log(10x^2 - 3x - 6)$.

   (d)   i. Show that $4\log_4 6 = 3\log_8 9 + 2$.

       ii. Find the equation of straight line passing through $(1, 3)$ and parallel to
       $2x - y + 3 = 0$.

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2. (a) Find the limit value of the following:

    i. $\lim_{x \to \sqrt{3}} \dfrac{x^2 - 3}{x - \sqrt{3}}$,

    ii. $\lim_{x \to \infty} \dfrac{7 - 3x^2}{4x^2 + 3x - 2}$,

    iii. $\lim_{x \to 2} \dfrac{\sqrt{x + 7} - 3}{x - 2}$.

(b) Differentiate the following with respect to x.

    i. $y = \ln\left(\dfrac{x^2 + 1}{x^2 - 1}\right)$,

    ii. $y = (x^2 + 3)^4$.

(c) Integrate the following:

    i. $\displaystyle\int x e^x \, dx$,

    ii. $\displaystyle\int \dfrac{\ln x}{x \sqrt{(\ln x)^2 + 2}} \, dx$.

(d) Find the maximum and minimum points of the function $y = 3x^2 - x^3$.