EASTERN UNIVERSITY SRI LANKA

FACULTY OF COMMERCE AND MANAGEMENT

FIRST YEAR, SECOND SEMESTER EXAMINATION IN BUSINESS ADMINISTRATION/ COMMERCE 2008/2009

(SEPT 2010) – PROPER AND REPEAT

COM 1032 BASIC CALCULUS

Answer All Questions **Time: 03 Hours**

01. (i) (a) If $f(x) = x + |x-2|$ compute $f(-1)$ and $f(3)$.

(b) Specify the domain of the function $g(t) = \sqrt{3t-2}$

(c) Find the composite function $f[g(1)]$ if $f(u) = \sqrt{u+1}$ and $g(x) = x^2 - 1$.

(d) A certain industrial machine depreciates so that its value after t years is given by a function $Q(t) = ke^{-0.04t}$. After 20 years the machine is worth 8986.58. Find the following:

(a) the value of k;

(b) original value of the machine.

(ii) A private college in the south west has launched a fund – raising campaign. Suppose that the college officials estimate that it will take $f(x) = \frac{10x}{150-x}$ weeks to reach x percent of their goal.

(a) What is the domain of the function?

(b) For what values of x does $f(x)$ have a practical interpretation in this context?

(c) Sketch the relevant portion of the graph of this function.

(d) How long will it take to reach 50 percent of the campaign's goal?

(20 Marks)

02. (i) Evaluate the limits of the functions given below:

(a) $\lim_{x \to -2} \frac{x^2+8}{x+2}$; (b) $\lim_{x \to 1} \frac{1-x}{\sqrt{5-x^2}-2}$.

(ii) Find $\frac{dy}{dx}$ for the functions given below.

(a) $y = \ln\left[\frac{\sqrt{4x+3}\,(x^2-2x+9)}{(3x-2)^{3/2}}\right]$; b) $y = \frac{e^{2x^2}+e^{3x+2}}{e^{3x}}$.

(iii) (a) If $x^2 + xy = 5$, find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Suppose that $y = \frac{1}{t}$ and $t = 3 - \frac{1}{x^2}$. Find the following:

(I) $\frac{dy}{dt}$; (II) $\frac{dt}{dx}$; (III) $\frac{dy}{dx}$ in terms of x and y;

(IV) $\frac{d^2y}{dx^2}$ at $x = 2$.

(iv) Suppose that the demand function is given by $q = 3p^2e^{5p^2+2p+6}$, where q is number of units and p is the price per unit. Find the elasticity of demand in terms of

(20 Mar

03. (i) Find relative maxima and minima and points of inflexion for the function

$$y = x^4(x-1) - \frac{1}{3}x^3$$

(ii) Suppose that the demand function is $x = \frac{1}{3}(25\text{-}2p)$, where x is the numbers of and p is the price per unit. Let the average cost per unit be Rs 40. Find:

(a) the revenue function in terms of p;
(b) the cost function in terms of p;
(c) the profit function;
(d) the price per unit that maximizes the profit function;
(e) the maximum profit.

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04. (i) (a) Find all the first and second order partial derivation for the function

$$f(x,y) = (x^3 + y^2)^2.$$

(b) Use the method of Lagrange multipliers to find the maximum values of f below subject to the given constraint:

$$f(x,y) = 4x^2 - 2xy + 6y^2; \quad x + y = 72$$

(ii) (a) The number of units of a product that are manufactured by a company is given by $f(k,L) = 10k^{0.4}L^{0.6}$, where k is the units of capital and L is the units of Labour.

(I) Find the marginal productivity of labour and capital ,

(II) Determine the effect on output of an additional unit of capital and labour at $k = 8, L = 20$.

(b) The profit function for a firm producing two goods x and y is given by $\pi(x,y) = 160x - 3x^2 - 2xy - 2y^2 - 120y - 18$. Find the profit maximizing level of output for each product and the maximum profit.

(20 Marks)

05. (i) Integrate the following

(a) $\int \left(\frac{1}{x^3} - \frac{x}{2}\right)^2 dx$; (b) $\int 2xe^x dx$.

(ii) Evaluate the following definite integrates

(a) $\int_0^3 \frac{6x}{x^2+1} dx$; (b) $\int_{-2}^3 e^{-x/2}$.

(iii) Marginal cost is given by $MC = 32 + 18q - 12q^2$. Fixed cost is 43. Find the total cost function.

(iv) The demand and supply function under perfect competition are $P_d = 16 - x^2$ and $P_s = 2x^2 + 4$ respectively. Find the consumer's surplus and producer's surplus.

(20 Marks)