

EASTERN UNIVERSITY, SRI LANKA

IRST EXAMINATION IN SCIENCE (2005/2006 & 2006/2007)

FIRST SEMESTER (Aug./Sep.'2007)

MT 106 - TENSOR CALCULUS

Answer all questions

Time: One hour

- 1. (a) Write the law of transformation for the tensors
 - i. Aik,
 - ii. B_{ijk}^{mn} ,
 - iii. Cm.
 - (b) Define the terms symmetric and skew-symmetric tensors.
 - i. $\Phi = a_{jk}A^jA^k$ show that we can always write $\Phi = b_{jk}A^jA^k$ where b_{jk} is symmetric.
 - ii. Show that the contraction of the outer product of the tensors A^p and B_q is an invariant.
 - (c) Find the covariant and contravariant components of a tensor in cylindrical coordinates (ρ, ϕ, z) , if its covariant components in rectangular coordinates are 2x z, x^2y , yz.

- (a) Define the following:
 - i. Christoffel symbols of first and second kind;
 - ii. Geodesics;
 - iii. Covariant derivative of B^i_{jk} and B_{pq} .
 - (b) With the usual notations, prove the followings:

i.
$$[pq, r] = [qp, r],$$

ii.
$$[pq,r]=g_{r,r}\Gamma_{pq}^{s}$$
,

iii.
$$\frac{\partial g^{pq}}{\partial x^m} = -g^{pn}\Gamma^q_{mn} - g^{qn}\Gamma^p_{mn}.$$

(c) Determine the christoffel symbols of second kind in sperical coordinate (r, θ, ϕ) and find the corresponding geodesic equations.

 $\Phi = \alpha_{j,k}A^jA^k$ show that we can always write $\Phi = b_{j,k}A^jA^k$ where $b_{j,k}$

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2), if its covariant components in rectangula

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