EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SECOND EXAMINATION IN SCIENCE - 2009/2010
FIRST SEMESTER (June/July, 2011)
MT207 - NUMERICAL ANALYSIS
(Proper & Repeat)

nswer all questions

Time : Two hours

1. (a) Define the following terms:
   i. absolute error;
   ii. relative error of a numerical value.

   (b) Evaluate $f(x) = x^3 - 6x^2 + 3x - 0.149$ at $x = 4.71$ using three digit arithmetic with chopping. Compute the absolute error and relative error.

   (c) Repeat the calculation in part (b), using the nesting form of $f(x)$ that was found in part (b). Calculate the relative error and compare with that found in part (b).

   (d) Describe what is meant by truncation error by reference to approximating $\sin x$ by x.

2. (a) Let $x = g(x)$ is the rearrangement of the equation $f(x) = 0$ and define the iteration,

$$x_{n+1} = g(x_n); \qquad n = 0, 1, 2... \tag{1}$$

with the initial value x_0. If $g'(x)$ exists, is continuous, and $|g'(x)| \leq K < 1$ for all x, then show that the sequence (x_n) generated by the iteration (1) converges to the unique root α of the equation $f(x) = 0$.

Show that the iteration, $x_{i+1} = \dfrac{2x_i - 3}{2 - x_i}$, have fixed points at $x = \pm\sqrt{3}$.

Hence investigate the convergence of the method.

(b) Obtain the Newton Raphson algorithm to compute the roots of the equations $f(x) = 0$ in an interval $[a, b]$.

Sketch the cubic polynomial $f(x) = 4x^3 - 10x^2 + 2x + 5$ to get a rough estimate of its roots. Use the Newton Raphson method to approximate each root to four decimal places.

3. (a) Construct a forward difference table for the following data.

x	1.0	1.5	2.0	2.5
$f(x)$	0.8988	0.9613	0.9945	0.9976

With $x_0 = 0.1$, estimate the approximation for the first derivative of $f(x)$ at $x = 1.5$ using the Newton's forward formula.

(b) Obtain the composite Trapezoidal rule to estimate $\int_a^b f(x)dx$ and derive a formula for error.

Let

$$I = \int_0^1 e^{-x^2}\,dx.$$

Estimate I using the composite Trapezoidal rule with 10 sub-intervals. Find an error bound in the elimination.

4. (a) Solve the following system of linear equations using the Gaussian Elimination with two digit rounding arithmetic and partial pivoting:

$$\begin{aligned} 2x_1 + 4x_2 - x_3 &= -5, \\ x_1 + x_2 - 3x_3 &= -9, \\ 4x_1 + x_2 + 2x_3 &= 9. \end{aligned}$$

(b) Find the solution of the following system of equations,

$$\begin{aligned} x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 &= \frac{1}{2}, \\ -\frac{1}{4}x_1 + x_2 - \frac{1}{4}x_4 &= \frac{1}{2}, \\ -\frac{1}{4}x_1 + x_3 - \frac{1}{4}x_4 &= \frac{1}{4}, \\ -\frac{1}{4}x_2 - \frac{1}{4}x_3 + x_4 &= \frac{1}{4}, \end{aligned}$$

using the Gauss-Seidel method and perform the first three iterations.