EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2010/2011

(EXTERNAL DEGREE)

SECOND SEMESTER (April/May, 2017)

EXTMT 105 - THEORY OF SERIES

(SPECIAL REPEAT)

Answer all questions

Time: One hour

1. Define what is meant by the convergent and divergent of an infinite series of real numbers $\sum_{k=1}^{\infty} a_k$.

   (a) A necessary condition for a series $\sum_{n=1}^{\infty} a_n$ to converge is that $\lim_{n\to\infty} a_n = 0$. Is it true that, it is a sufficient condition for the convergence of the series $\sum_{n=1}^{\infty} a_n$? Justify your answer.

   Does the following series converge or diverge? Explain your answer.

$$\sum_{n=1}^{\infty} \frac{n(n+1)}{\sqrt{n^3+2n^2}}$$

[50 marks]

   (b) Prove that the geometric series

$$\sum_{n=1}^{\infty} ar^{n-1}$$

   converges if $|r| > 1$ and diverges otherwise. Where a and r are real constants. Does the following series converge or diverge? If it converge, find the sum. If it diverge, explain why?

$$\sum_{n=1}^{\infty}\left(\frac{3^n}{6^n}+\frac{2^n}{6^n}\right).$$

[50 ma

2. (a) i. Check the convergence of the following series by using the limit for comparison test

$$\sum_{n=1}^{\infty}\frac{n^2+n^3}{n^4+1}.$$

[25 ma

ii. Use the root test to determine whether the series

$$\sum_{n=1}^{\infty}\frac{1}{[\ln(n+1)]^n}$$

converges or diverges. [20 ma

iii. Investigate whether the following series is convergent or divergent using the alternating series test

$$\sum_{n=2}^{\infty}\frac{\cos(n\pi)}{\sqrt{n}}.$$

[25 ma

(b) Find the radius of convergence and interval of convergence of the follow power series.

$$\sum_{n=0}^{\infty}\frac{(x-4)^n}{5^n}.$$

[30 mar