EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
FIRST EXAMINATION IN SCIENCE - 2010/2011
FIRST SEMESTER (November, 2012)
MT 101 - FOUNDATION OF MATHEMATICS
(Re-repeat)

Answer all questions

Time : Three hours

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1. (a) Let p and q be two statements such that $p \to q$ is false. Find the truth value of each of the following statements:

   i. $\sim q \to p$;

   ii. $p \wedge (q \to \sim p)$;

   iii. $\sim (\sim p \wedge q) \wedge q$.

   (b) Using the laws of algebra of logic, prove the following logical equivalences:

   i. $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$;

   ii. $(p \wedge \sim q) \vee (p \wedge q) \equiv p$.

   (c) Using the valid argument forms, deduce the conclusion $\sim t \vee r$ from the premises given below:

   $$\sim p \to r \wedge \sim s$$
   $$t \to s$$
   $$u \to \sim p$$
   $$\sim w$$
   $$u \vee w.$$

2. Let A, B and C be subsets of a universal set X.

(a) Simplify the following expressions:

i. $[(A \cup \Phi) \cap (B \cup A') \cap (A \cup B \cup X)]'$;

ii. $[A \cup (B \cap C)] \cup [(B' \cap C') \cup C]'$.

(b) Prove that if $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then $B = C$. Hence show that $A \backslash (B \cup C) = (A \backslash B) \cup (A \backslash C)$ if and only if $A \Delta (B \cup C) = (A \Delta B) \cup (A \Delta C)$.

3. (a) Let $f : X \rightarrow X$ be a function and define a relation R on X by $xRy \Leftrightarrow y = f(x)$. Prove the following

i. R is reflexive $\Leftrightarrow f = I$;

ii. R is symmetric $\Leftrightarrow f \circ f = I$;

iii. R is transitive $\Leftrightarrow f \circ f = f$;

where I denotes the identity mapping from X to X.

(b) If $f : A \rightarrow B$ is a mapping, prove that the relation R_f defined on Y by $xR_fy \Leftrightarrow f(x) = f(y)$ is an equivalence relation.

Let $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} (x/\sqrt{(x^2+y^2)}, y/\sqrt{(x^2+y^2)}), & \text{if } (x,y) \neq (0,0); \\ (0,0), & \text{if } (x,y) = (0,0). \end{cases}$$

Find the R_f class of (0,1).

4. (a) Define each of the following terms:

i. injective mapping;

ii. surjective mapping.

(b) Let p be a fixed positive integer. Prove that the mapping $f : \mathbb{Z} \rightarrow \mathbb{Z}$ given by

$$f(n) = \begin{cases} n+p, & \text{if } n \text{ is divisible by } p, \\ n, & \text{if } n \text{ is not divisible by } p; \end{cases}$$

is a bijection. Determine f^{-1}.

(c) Let $f : X \rightarrow Y$ be a mapping. Prove that f is injective iff $f(A \cap B) = f(A) \cap f(B)$ for all subsets A, B of X.

5. (a) Show that every partially ordered set has at most one first element.

(b) Show that first element of every partially ordered set is a minimal element. Is the converse true? Justify your answer.

(c) Prove that in a totally ordered set every minimal element is a first element.

6. (a) For any integer a, prove that:

i. $2 \mid a(a+1)$ and $3 \mid a(a+1)(a+2)$;

ii. $3 \mid a(2a^2+7)$;

iii. if a is odd then $32 \mid (a^2+3)(a^2+7)$.

(b) When Mr. Smith cashed a cheque in a Bank, the teller mistook the number of cents for the number of rupees and vice versa. Unaware of this, Mr. Smith spent 68 cents and noticed to his surprise that he had twice the amount of the original cheque. Determine the smallest value for which the cheque could have been written.