



**EASTERN UNIVERSITY, SRI LANKA**

**DEPARTMENT OF MATHEMATICS**

**SECOND EXAMINATION IN SCIENCE - 2010/2011**

**FIRST SEMESTER (March/April, 2013)**

**AM 207 - NUMERICAL ANALYSIS**

**(Proper & Repeat)**

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Answer all questions

Time : Two hours

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1. (a) Define the terms:

- i. absolute error;
- ii. relative error of a numerical value.

(b) Let  $f(x) = \sin 2x + \cos 2x$ .

- i. Write down the second Taylor polynomial  $P_2(x)$  of  $f(x)$  centered around  $x_0 = 0$ .
- ii. Write down the corresponding Taylor remainder  $R_2(x)$ .
- iii. Suppose  $P_2(x)$  is used to approximate  $f(x)$  on the interval  $[-\pi, \pi]$ . How large in magnitude can the absolute error in this approximation be?

2. (a) Let  $x = \phi(x)$  be the rearrangement of the equation  $f(x) = 0$  and define the iteration,

$$x_{n+1} = \phi(x_n), \quad n = 0, 1, \dots,$$

with the initial value  $x_0$ . If  $\phi'(x)$  exists and is continuous such that  $|\phi'(x)| \leq K < 1$  for all  $x$ , then show that the sequence  $(x_n)$  generated by the above iteration converges to the unique root  $\alpha$  of the equation  $f(x) = 0$ .

Find a real root of the equation

$$f(x) = x^3 + x^2 - 1 = 0$$

by the method of iteration.

- (b) i. Define the order and the asymptotic error constant of the iteration method to compute the non linear equation  $f(x) = 0$ .
- ii. Show that the order of convergence of secant method is approximately 1.62.

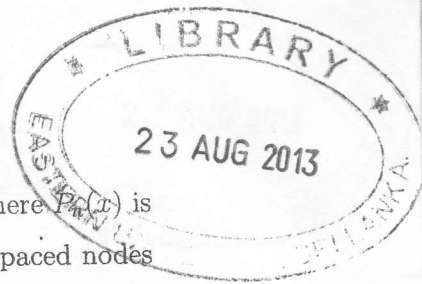
Apply secant method to find a solution to the equation  $x - \cos x = 0$  near  $x = 0$  in the interval  $\left[0, \frac{\pi}{2}\right]$  that is accurate within  $10^{-4}$ .

3. (a) Let  $f(x)$  be a  $(n + 1)$ -times continuously differentiable function of  $x$  and  $y_0, y_1, \dots, y_n$  be the values of  $f(x)$  at  $x = x_0, x_1, \dots, x_n$ , respectively. Then derive the Lagrange's Interpolation polynomial  $P_n(x)$  to estimate the value of  $f(x)$  for any  $x \in [x_0, x_n]$ , in the form

$$P_n(x) = \sum_{i=0}^n \frac{\Pi(x)y_i}{(x - x_i)\Pi'(x_i)},$$

where  $\Pi(x) = \prod_{i=0}^n (x - x_i)$ .

- (b) Consider the exponential function  $y(x) = e^x$ . Use the Lagrange interpolating polynomial to estimate  $e^{0.2}$ , using  $y(x)$  at the nodes  $(0, 0.1, 0.3)$ . Estimate the maximum error in your approximation to  $e^{0.2}$ .



4. (a) Let  $I(f) = \int_a^b f(x)dx$  and  $I(P_n)$  is the approximation to  $I(f)$ , where  $P_n(x)$  is the interpolating polynomial which interpolates  $f(x)$  at equally spaced nodes  $a = x_0, x_1, \dots, x_n = b$ , where  $x_k = x_0 + kh$ , for  $k = 0, 1, \dots, n$ , and  $x_i \in [a, b]$  for  $i = 0, 1, \dots, n$ . Then the error in the approximation is given by

$$E(f) = I(f) - I(P_n).$$

Obtain the composite Trapezoidal rule and show that the composite error is

$$-\frac{(b-a)}{12}h^2y''(\xi), \text{ where } \xi \in [a, b].$$

Determine

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+x} dx$$

to an accuracy of  $\epsilon = 10^{-2}$ . using the Composite Trapezium Rule.

- (b) Solve the following linear system of equations using Gaussian Elimination with partial pivoting:

$$\begin{aligned} 4x_1 + 4x_2 + x_3 + 4x_4 &= 12 ; \\ 2x_1 + 5x_2 + 7x_3 + 4x_4 &= 1 ; \\ 10x_1 + 5x_2 - 5x_3 &= 25 ; \\ -2x_1 - 2x_2 + x_3 - 3x_4 &= -10 . \end{aligned}$$