



EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST YEAR EXAMINATION IN SCIENCE - 2016/2017

FIRST SEMESTER - (AUG./SEPT., 2018)

MT 1022 - ORDINARY DIFFERENTIAL EQUATIONS

Answer All Questions

Time Allowed: 2 Hours

- Q1. (a) State the necessary and sufficient condition for the ordinary differential equation (ODE)

$$M(x, y) dx + N(x, y) dy = 0$$

to be *exact*.

[10 Marks]

Find the general solution of the ODE

$$[xy \sin(xy) - \cos(xy) - e^{2x}] dx - [y^2 - x^2 \sin(xy)] dy = 0.$$

[60 Marks]

- (b) Solve the nonlinear first-order Bernoulli's equation

$$x \frac{dy}{dx} + 2y - x^3 y^3 = 0.$$

[30 Marks]

- Q2. If $y_1 = x$ is a particular solution of the nonlinear Riccati equation

$$\frac{dy}{dx} - x^3 y^2 - \left(\frac{1}{x} - 2x^4 \right) y - x^5 = 0$$

then obtain the general solution of the equation.

[100 Marks]

- Q3. (a) Let $D \equiv d/dx$ be a differential operator. Obtain the general solution of the ODE

$$(D^3 - D^2 - 2D)y = 44 - 76x - 48x^2.$$

[50 Marks]

- (b) Find the general solution of the Cauchy-Euler differential equation

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} = \sin(\ln x) + x.$$

[50 Marks]

- Q4. (a) Define what is meant by *orthogonal trajectories* of curves.

[10 Marks]

Find the orthogonal trajectories of the family of curves

$$r = a(1 + \cos \theta)$$

in polar coordinates, where a is a constant. [40 Marks]

- (b) Find the general solution of the simultaneous ODEs

$$\begin{aligned}(D^2 + D + 1)x + (D^2 + 1)y &= e^{2t} \\ (D^2 + D)x + D^2 y &= e^{-t},\end{aligned}$$

where $D \equiv d/dt$.

[50 Marks]
