EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE - 2016/2017

SECOND SEMESTER (May/June, 2019)

MT 1042 - VECTOR ANALYSIS

Answer all questions | Time: Two hours

1. (a) Find the equation of the plane passing through three given terminal points of $\underline{a}$, $\underline{b}$ and $\underline{c}$.

   (b) Prove that the diagonals of a parallelogram bisect each other.

   (c) Find $\underline{x}$ in terms of $\underline{a}$ and $\underline{b}$ if $\underline{x} \wedge \underline{a} = \underline{b} - \underline{x}$.

   (d) Prove that the radius of curvature (ρ) of the curve with parametric equations $x = x(s)$, $y = y(s)$ and $z = z(s)$ is

$$\rho = \left[\left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 + \left(\frac{d^2z}{ds^2}\right)^2\right]^{1/2},$$

   where s is an arc length of the curve.

2. (a) Define the following terms:

   (i) the gradient of a scalar field ϕ;

   (ii) the curl of a vector field $\underline{\mathrm{A}}$.

   (b) Prove that if ϕ is a scalar field and $\underline{A}$ is a vector field, then

$$\mathrm{curl}(\phi\underline{\mathrm{A}}) = \phi \ \mathrm{curl}\ \underline{\mathrm{A}} + \mathrm{grad}\ \phi \wedge \underline{\mathrm{A}}.$$

   (c) Let $\underline{a}$ be a non-zero constant vector and $\underline{r}$ be a position vector of a point. If $r = |\underline{r}|$, find grad $\left(\frac{\underline{\mathrm{a}} \cdot \underline{\mathrm{r}}}{\mathrm{r}^5}\right)$.

   Hence show that

$$\mathrm{curl}\left[\left(\frac{\underline{\mathrm{a}} \cdot \underline{\mathrm{r}}}{\mathrm{r}^5}\right)\underline{\mathrm{r}}\right] = \frac{\underline{\mathrm{a}} \wedge \underline{\mathrm{r}}}{\mathrm{r}^5}.$$

(d) Find the unit normal vector to the surface $x^4 - 3xyz + z^2 + 1 = 0$ at the point $(1, 1, 1)$.

3. (a) Define the following terms:

   (i) conservative vector field;

   (ii) solenoidal vector field.

   (b) Let $\underline{A} = (y^2 \cos x + z^3)\underline{i} + (2y \sin x - 4)\underline{j} + (3xz^2 + 2)\underline{k}$.

   i. Show that $\underline{A}$ is a conservative vector field but not solenoidal.

   ii. Find the scalar potential ϕ such that $\underline{A} = \underline{\nabla}\phi$.

   (c) Find the directional derivative of $\phi = xy + yz + zx$ in the direction of the vector $2\underline{i} + 3\underline{j} + 6\underline{k}$ at the point $(3, 1, 2)$.

4. (a) If $\underline{A} = (2y + 3)\underline{i} + xz\underline{j} + (yz - x)\underline{k}$, then evaluate the integral $\int_C \underline{A} \cdot d\underline{r}$, where C is the straight line path from $(0, 0, 0)$ to $(0, 0, 1)$ then to (0,1,1) and then to (2,1,1).

   (b) State the Divergenge theorem and use it to evaluate $\iiint_V \underline{\nabla} \cdot \underline{A} \, dV$, where $\underline{A} = z\underline{i} + x\underline{j} - 2y^2z\underline{k}$ and V is the volume of the cylinder $x^2 + y^2 = 16$ include in the first octant between the planes $z = 0$ and $z = 5$.