EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
FIRST YEAR EXAMINATION IN SCIENCE - 2016/2017
SECOND SEMESTER - (APRIL/MAY, 2019)
MT 1232 - MATHEMATICAL METHODS

Answer **All** Questions Time Allowed: **2** Hours

Q1. (a) Define what is meant by the point, $x = x_0$, being

(i) an *ordinary* ;

(ii) a *singular*;

(iii) a *regular singular*

point of the ordinary differential equation (ODE)

$$y'' + p(x)y' + q(x)y = 0,$$

where the prime denotes differentiation with respect to x, and $p(x)$ and $q(x)$ are rational functions.

[30 Marks]

(b) (i) Find the regular singular point(s) of the ODE

$$4xy'' + 2y' - 7y = 0. \qquad (1)$$

(ii) Use the method of Frobenius to find the general solution of the equation (1).

[70 Marks]

Q2. (a) Find the general solution of the linear first-order partial differential equation (PDE)

$$(x^2 + y^2 - yz)\frac{\partial z}{\partial x} - (x^2 + y^2 - xz)\frac{\partial z}{\partial y} = z(x - y).$$

[40 Marks]

(b) Find the integral surface of the PDE

$$x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} + z^2 = 0$$

which passes the hyperbola

$$xy = x + y, \quad z = 1.$$

[60 Marks]

Q3. (a) (i) Define the *gamma-function* $\Gamma(x)$ and *beta-function* $B(m, n)$, where m, n are positive integers.

(ii) Evaluate the integral

$$\int_0^1 \frac{dx}{\sqrt{1-x^4}}.$$

(You may use the following results without proof

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.)$$

[40 Marks]

(b) Find the solution of the PDE using separation of variables

$$\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = (x + 2y)u.$$

[60 Marks]

Q4. Let $x = x(t)$, $y = y(t)$, and let the prime denotes differentiation with respect to t.

(a) Solve each of the following initial value problems (IVPs) using the Laplace transforms:

(i) $x'' - 3x' + 2x = 4e^{2t}, \quad x(0) = -3, \quad x'(0) = 5;$

(ii) $x''' - 3x'' + 3x' - x = t^2e^t, \quad x(0) = 1, \quad x'(0) = 0, \quad x''(0) = -2.$

[60 Marks]

(b) Solve the system of the simultaneous ODEs by using the Laplace transforms:

$$x' = x - 2y,$$
$$y' = 5x - y,$$

subject to the initial conditions; $x(0) = -1, \ y(0) = 2.$

[40 Marks]

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