# EASTERN UNIVERSITY, SRI LANKA

## THIRD EXAMINATION IN SCIENCE-2010/2011

## FIRST SEMESTER (April, 2013)

## MT304 - GENERAL TOPOLOGY

**Answer all questions** **Time : Two hours**

1. Define the following terms:

   - Topology on a set;
   - Interior of a set.

   (a) Let $X$ be a non-empty set. Let $\tau$ be the collection of subsets of $X$ containing the empty set $\Phi$ and all subsets whose complements are finite. Is $(X, \tau)$ a topological space? Justify your answer.

   (b) Let $A$ be a non-empty subset of a topological space $(X, \tau)$. Prove that

   i. the interior of $A$ is the largest open set contained in $A$.

   ii. $A$ is open if and only if $A = A^{\circ}$.

   (c) Let $X = \{1, 2, 3\}$ and $\tau = \{X, \Phi, \{1, 2\}, \{2, 3\}, \{2\}\}$. Let $A = \{1, 2\}$. Find the interior of $A$.

2. (a) If $(X, \tau)$ is a topological space, where $\tau = \{A \subseteq X \mid A = \Phi \text{ or } A^c \text{ is finite}\}$ and $X$ is an infinite set. Prove that $\overline{A} = X$ for any infinite subset $A$ of $X$.

(b) Let $(Y, \tau_Y)$ be a subspace of a topological space $(X, \tau)$. Prove that $A \subseteq Y$ is a closed subset of $Y$ in $(Y, \tau_Y)$ if and only if $A = F \cap Y$ for some closed subset $F$ of $X$ in $(X, \tau)$.

(c) Let $f$ be a function from a topological space $(X, \tau_1)$ into a topological space $(Y, \tau_2)$.

i. Prove that, $f$ is continuous on $X$ if and only if $f^{-1}(G)$ is open in $X$ for every open subset $G$ in $Y$.

ii. Prove that, $f$ is continuous on $X$ if and only if $f^{-1}(A^\circ) \subseteq \{f^{-1}(A)\}^\circ$ for every subset $A$ of $Y$.

3. Let $(X, \tau)$ be a topological space. Prove that the following statements are equivalent:

(i) $X$ is connected;

(ii) $X$ cannot be expressed as the union of two disjoint non-empty closed sets;

(iii) The only subsets of $X$ which are both open and closed are $X$ and $\Phi$;

(iv) The set of all frontier points of $A$, denoted by Fr $A$, is non-empty, for any non-empty proper subset $A$ of $X$;

(v) There is no continuous function from $X$ onto $Y$, when $Y = \{0, 1\}$ has the discrete topology.

4. Define the following terms:

- Frechet space ($T_1$);
- Housdorff space ($T_2$);
- Compact set.

(a) Prove that a closed subset of a compact topological space is compact.

(b) Prove that every compact subset of a Housdorff topological space is closed.

(c) Prove that every Housdorff space is a Frechet space. Is the converse true? Justify your answer.