EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2008/2009
THIRD YEAR SECOND SEMESTER (Feb./Apr., 2015)
EXTMT 307 - CLASSICAL MECHANICS III
PROPER & REPEAT

Answer all Questions

Time: Three hours

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1. Two frames of reference S and S' have a common origin O, and S' rotates with an angular velocity $\underline{\omega}$ relative to S. If a moving particle P has its position vector $\underline{r}$ relative to O at time t, show that :

   (a) $\dfrac{d\underline{r}}{dt} = \dfrac{\partial \underline{r}}{\partial t} + \underline{\omega} \wedge \underline{r}$, and

   (b) $\dfrac{d^2\underline{r}}{dt^2} = \dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \dfrac{\partial \underline{r}}{\partial t} + \dfrac{\partial \underline{\omega}}{\partial t} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r})$.

   An object of mass m initially at rest is dropped to the earth's surface from a hight h above the earth's surface. Assume that the angular speed of the earth about its axis is a constant ω. Prove that after time t the object is deflected east of the vertical by the amount

$$\frac{1}{3}\omega g t^3 \cos\lambda \ ,$$

   where λ is the earth's latitude.

2. (a) Define what is meant by the following terms:

   i. *linear momentum*;

   ii. *angular momentum*;

   iii. *moment of force.*

(b) A solid of mass M is in the form of a tetrahedron $OXYZ$, the edges OX, OY, OZ are mutually perpendicular, rests with XOY on a fixed smooth horizontal plane and YOZ against a smooth vertical wall. The normal to the rough face XYZ is in the direction of a unit vector $\underline{n}$. A heavy uniform sphere of mass m and center C rolls down the face causing the tetrahedron to acquire a velocity $-V\underline{j}$ where $\underline{j}$ is the unit vector along OY. If $\overrightarrow{OC} = \underline{r}$, then prove that

$$(M+m)V - m\dot{\underline{r}}\cdot\underline{j} = \text{constant},$$

and that

$$\frac{7}{5}\ddot{\underline{r}} = \underline{f} - \underline{n}(\underline{n}\cdot\underline{f}),$$

where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and $\underline{g}$ is the acceleration of gravity.

3. With the usual notation, obtain the Euler's equations for the motion of the rigid body having a point fixed in the form:

$$A\dot{\omega}_1 - (B-C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = N_2,$$

$$C\dot{\omega}_3 - (A-B)\omega_1\omega_2 = N_3.$$

A body moves about a point O under no forces. The principle moment of inertia at O being $3A$, $5A$ and $6A$. Initially the angular velocity has components $\omega_1 = n$, $\omega_2 = 0$ and $\omega_3 = n$ about the corresponding principal axes. Show that at any time t,

$$\omega_2 = \frac{3n}{\sqrt{5}}\tan\left(\frac{nt}{\sqrt{5}}\right).$$

4. With the usual notations, derive Lagrange's equation for the impulsive motion from Lagrange's equations for a holonomic system in the following form

$$\triangle\left(\frac{\partial T}{\partial \dot{q}_j}\right) = S_j\,, \qquad j = 1, 2, ..., n.$$

A uniform rod AB of length $2a$ and mass m has a particle of mass M attached to the end B. It is at rest on a smooth horizontal table when an impulse I is applied at A in a direction perpendicular to AB, and in the plane of the table. Find the initial velocities of A and B and prove that the resulting kinetic energy is

$$\frac{2I^2(m+3M)}{m(m+4M)}.$$

5. (a) Define the *Hamiltonian function* in terms of the Lagrangian function . Show with the usual notations that the Hamiltonian equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial p_j},\ \dot{p}_j = -\frac{\partial H}{\partial q_j} \text{ and } \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$

(b) Prove that if the time t does not occur in the Lagrangian function L, then the hamiltonian function H is also not involved in t.

(c) Write down the Hamiltonian, and then find the equation of motion when the particle of mass m is moving on a cartesian coordinate system.

6. (a) Define what is meant by the *Poisson bracket*.
Show that the Hamiltonian equations of the holonomic system may be written in the form

$$\dot{q}_k = [q_k, H]\,, \quad \dot{p}_k = [p_k, H]\,,$$

and show that for any function $f(q_i,\ p_i,\ t)$,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f,\ H]\,,$$

where H is a Hamiltonian function.

(b) Show that, if f and g are constants of motion then their poisson bracket $[f, g]$ is also a constant of motion.