EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE ~~2008/2009~~ (2010/2011)

THIRD YEAR FIRST SEMESTER (Mar./May, 2017)

EXTMT 304 - GENERAL TOPOLOGY (REPEAT)

Answer all questions **Time : Two hours**

---

1. Define the following terms:

   - topology on a set;
   - subspace of a topology;
   - base for a topology.

   (a) Let X be a non-empty set and let τ be the collection of subsets of X consisting of the empty set Φ and all subsets whose complements are finite. Prove that τ is a topological on X.

   (b) Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that $A \subseteq Y$ is closed in (Y, τ_Y) if and only if $A = F \cap Y$ for some closed subset F of X in (X, τ).

   (c) Let $\mathbb{B}$ be a base for a topology τ on X and let $S \subseteq X$. Show that the collection $\mathbb{B}_S = \{U \cap S \mid U \in \mathbb{B}\}$ is a base for the relative topology τ_S on S.

2. (a) Let A and B be two non-empty subsets of a topological space (X, τ).

   Prove the following:

   i. an interior of A , (A°), is the largest open subset of A;

   ii. the closure of A , $(\overline{A})$, is the smallest closed set containing the set A;

   iii. $(A \cap B)^\circ = A^\circ \cap B^\circ$.

   (b) Let a function f from a topological space (X, τ) into another topological space $(Y,$ be continuous. Prove that for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$.

3. Define the term "disconnected set" in a topological space.

   Prove the following:

   (a) a topological space (X, τ) is disconnected if and only if there exist a non em proper subset of X which is both open and closed.

   (b) a topological space (X, τ) is disconnected if and only if there exist a non-em proper subset of A of X such that $Fr(A) = \Phi$.

   (c) continuous image of a connected set is connected.

4. Define the following terms:

   - Frechet space (T_1);
   - Housdorff space (T_2);
   - Compact set.

   (a) Prove that every subset of a co-finite topological space is compact.

   (b) Prove that continuous image of a compact set in a topological space is compact.

   (c) Prove that every Hausdorff space is a Frechet space.

   Is the converse true? Justify your answer.