EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
THIRD YEAR EXAMINATION IN SCIENCE - 2010/2011
SECOND SEMESTER (March/May,2017)
EXTMT 307 - CLASSICAL MECHANICS
SPECIAL REPEAT

Answer all Questions **Time: Three hours**

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1. Two frames of reference S and S' have a common origin O and S' rotates with an constant angular velocity $\underline{\omega}$ relative to S. If a moving particle P has its position vector as $\underline{r}$ relative to O at time t, show that :

   (a) $\dfrac{d\underline{r}}{dt} = \dfrac{\partial \underline{r}}{\partial t} + \underline{\omega} \wedge \underline{r}$, and

   (b) $\dfrac{d^2\underline{r}}{dt^2} = \dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \dfrac{\partial \underline{r}}{\partial t} + \dfrac{\partial \underline{\omega}}{\partial t} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r})$.

   An object is thrown downward with an initial speed v_0. Prove that after time t the object is deflected east of the vertical by the amount

   $$\omega v_0 \sin\lambda t^2 + \frac{1}{3}\omega g \sin\lambda t^3,$$

   where λ is the earth's co - latitude.

2. (a) With the usual notations, obtain the equations of motion for a system of N particles in the following forms:

   i. $M\underline{f}_G = \sum_{i=1}^{N} \underline{F}_i$,

   ii. $\frac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i$,

   where $\sum_{i=1}^{N} \underline{h}_i = \underline{H}$ and $\underline{h}_i = \underline{r}_i \wedge m_i \underline{v}_i$.
   (State clearly the results that you may use)

   (b) A solid of mass M is in the form of a tetrahedron $OXYZ$, the edges OX, OY, OZ of which are mutually perpendicular, rests with XOY on a fixed smooth horizontal plane and YOZ against a smooth vertical wall. The normal to the rough face XYZ is in the direction of a unit vector $\underline{n}$. A heavy uniform sphere of mass m and center C rolls down the face causing the tetrahedron to acquire a velocity $-V\underline{j}$ where $\underline{j}$ is the unit vector along OY
   If $\overrightarrow{OC} = \underline{r}$, then prove that

$$(M+m)V - m\dot{\underline{r}} \cdot \underline{j} = \text{constant}$$

   and that

$$\frac{7}{5}\,\ddot{\underline{r}} = \underline{f} - \underline{n}(\underline{n} \cdot \underline{f}) ,$$

   where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and $\underline{g}$ is the acceleration of gravity.

3. (a) With the usual notations obtain the *Euler's* equations for the motion of the rigid body, having a point fixed, in the form:

$$A\dot{\omega}_1 - (B-C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = N_2,$$

$$C\dot{\omega}_3 - (A-B)\omega_1\omega_2 = N_3.$$

   A body moves about a point O under no forces. The principle moment of inertia at O being $3A$, $5A$ and $6A$. Initially the angular velocity has components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = 3$ about the corresponding principal axes. Show that at time t,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tan\left(\frac{nt}{\sqrt{5}}\right).$$

4. With the usual notations, derive the Lagrang's equation for the impulsive motion from the Lagrange's equations for a holonomic system in the following form

$$\triangle \left(\frac{\partial T}{\partial \dot{q}_j} \right) = S_j \quad \text{for} \quad j = 1, 2, ..., n.$$

A uniform rod AB of length l and mass m is at rest on a horizontal smooth table. An impulse of magnitude I is applied to one end A in the direction perpendicular to AB. Prove that, immediate after the application of impulse,

(a) the one end A of the rod AB has the velocity of magnitude $\frac{4I}{m}$,

(b) center of mass of the rod AB has the velocity of magnitude $\frac{I}{m}$,

(c) the rod rotates about the center of mass with angular velocity of magnitude $\frac{6I}{m}$.

5. (a) Define *Hamiltonian* function in terms of Lagrangian function .

Show that, with the usual notations, that the Hamiltonian equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \ \dot{p}_j = -\frac{\partial H}{\partial q_j} \quad \text{and} \quad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$

(b) Prove that if the time t does not occur in Lagrangian function L, then the Hamiltonian function H is also not involved in t.

(c) Write down the Hamiltonian function H and then find the equation of motion for a simple pendulum.

6. (a) Define the *poisson bracket.*

Show that for any function $f(q_i, \ p_i, \ t)$,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, \ H],$$

where H is a Hamiltonian function.

(b) With the usual notations, prove that:

i. $\frac{\partial}{\partial t}[f, \ g] = \left[\frac{\partial f}{\partial t}, \ g\right] + \left[f, \ \frac{\partial g}{\partial t}\right];$

ii. $[f, \ q_k] = -\frac{\partial f}{\partial p_k};$

iii. $[f, \ p_k] = \frac{\partial f}{\partial q_k}.$

(c) Show that, if f and g are constants of motion then their poisson bracket $[f, \ g]$ is also a constant of motion.