

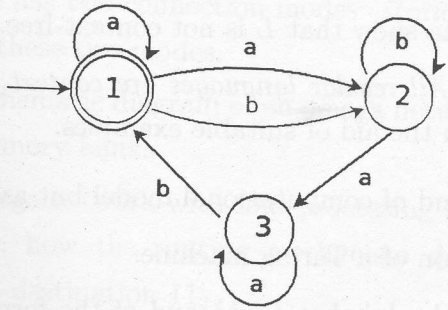
Time allowed: Three Hours

Finite automata are good models for computers with an extremely limited amount of memory.

- a) State the formal definition of a finite automaton. [20%]
- b) Create a finite automaton for strings made of "0" and "1" with a minimum length 3 and having every even character as "1". [30%]
- c) Draw the state diagram for the finite automaton you have created in part (b). [10%]
- d) Describe what is meant by *regular language*. [20%]
- e) Differentiate *Deterministic Finite Automata* and *Non-deterministic Finite Automata*. [20%]

Regular expressions are another means to define languages.

- a) State the advantages of regular expressions over finite automaton. [10%]
- b) Write a regular expression for the language with alphabet "0" & "1" and contains an even number of "0"s and each "0" is followed by at least one "1". [25%]
- c) Convert the finite automaton given in the following figure into regular expression. [25%]



- a) State the *Pumping lemma* for regular languages and its purpose. [20%]
- b) Let  $L$  be a language with alphabet  $\{a, b\}$  and of the form  $\{a^n b^{2n} : n \geq 0\}$ . Prove by applying the pumping lemma that  $L$  is not regular. [20%]

Context-free grammars are traditionally used for defining the syntax of programming languages and their compilation.

- a) State the definition of a context free grammar. [25%]
- b) Let  $L$  be a language of strings  $w$  with alphabet  $\{a, b\}$  satisfying the following:
  - there are no empty strings
  - the number of "a"s in  $w$  is equal to the number of "b"s in  $w$
  - $w$  does not contain the substrings "abba" and "bbaa"

[Question 3 continues on the next page]

- i. Write five different valid strings of the above language.
  - ii. Create a context free grammar for the language  $L$  defined above. Note: You should be able produce as many different patterns of strings as possible possible valid strings.
  - iii. Create a parse tree for a string with at least eight alphabets. Note: Choose such that it needs many grammar rules for parsing.
4. *Pushdown automaton* is another kind of computational model similar like *finite automaton*.
  - (a) Give the definition of a *deterministic pushdown automaton*.
  - (b) Write the format of the *transition function* of the pushdown automaton clearly.
  - (c) Let  $L$  be a language of strings  $w$  with alphabet  $\{a, b\}$  and of the form  $\{a^n b^n : n \geq 1\}$ . Create a pushdown automaton for the language  $L$ .
  - (d) Apply the pushdown automation developed in part (c) to show that the string  $a^n b^n$  is a valid string of the language  $L$  defined in part (c).
5. Pumping lemma for regular languages can be generalised for *context-free languages*.
  - (a) Write the Pumping lemma for the context-free Languages.
  - (b) Let  $L$  be a language of strings  $w$  with alphabet  $\{a, b\}$  and of the form  $\{a^n b^n : n \geq 1\}$ . Using the pumping lemma show that  $L$  is not context-free.
  - (c) Explain the statement “*All regular languages are context-free but not all context-free languages are regular*”, with the aid of suitable examples.
6. *Turing machine* is another kind of computational model but as powerful as a *pushdown automaton*.
  - (a) Write the formal definition of a Turing machine.
  - (b) Let  $L$  be a language with alphabet  $\{a, b\}$  and of the form  $\{a^n b^{2n} : n \geq 1\}$ . Construct a Turing machine with one tape that can accept strings of the language  $L$ .
  - (c) Show how the Turing machine constructed in part (b) works to accept the string  $a^n b^{2n}$ .