EASTERN UNIVERSITY, SRI LANKA

SPECIAL DEGREE EXAMINATION IN SCIENCE – 2011/2012

(SEPTEMBER/OCTOBER - 2016)

PHS 02 ADVANCED QUANTUM MECHANICS

03 hour

er ALL Questions

(a) Show that the one-dimension Schrödinger equation for a particle of mass m, subjected to a restoring force kx can be written as,

$$\frac{d^2\psi(y)}{dy^2}+(\lambda-y^2)\psi(y)=0$$

where $\lambda=\frac{2E}{\hbar}\sqrt{\frac{m}{k}}$ and $y=\alpha x=\left[\frac{km}{\hbar}\right]^{\frac{1}{4}}x$, E being the energy of the particle.

(b) Writing $\psi(y)=H(y)e^{-y^2/2}$, where $H(y)$ is a polynomial in y, show that the above equation reduces to the form,

$$\frac{d^2H(y)}{dy^2}-2y\frac{dH(y)}{dy}+(\lambda-1)H(y)=0.$$

(c) The ground state wave function of a harmonic oscillator is given by,

$$\psi_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(\frac{m\omega x^2}{2\hbar}\right)$$

(i) where is probability density maximum?

(ii) what is the value of the maximum probability?

The wave function of a particle confined to an infinite one-dimensional potential well of width L (between $x=0$ and $x=L$) is given by,

$\psi_0(x)=A\sin(kx)$ where $k=\sqrt{\frac{2mE}{\hbar^2}}$, E is the energy of the particle, m is the mass and A is a constant.

(a) Applying the necessary boundary conditions, show that the energy of the pa[rticle is] quantized. Find the quantized energy values and corresponding normalize[d wave] functions of the particles.

(b) Draw the wave functions of the ground state, and the first exited state of the p[article].

(c) For the case $m = 0.06\ m_e$ and $L = 50$ Å, where m_e is the mass of the [electron,] calculate the wavelength of photons necessary to exit the particle from the [ground] state to the first exited state.

($m_e = 9.1\times10^{-31}$ kg, $h = 6.6\times10^{-34}$ Js $c = 3\times10^{8}$ ms^{-1})

(d) If the potential outside the well is finite, sketch the form of the new gro[und state] wave function inside and the outside the well without further calculations.

03. (a) Show that

i. the eigenvalues of a Hermitian operator are real.

ii. the eigenstates corresponding to different eigenvalues of a Hermitian op[erator are] orthogonal.

(b) Check whether the following operators are Hermitian or not and [find] eigenvalues to verify a (i). [Hint: use the matrix representation of operators]

i. $\hat{O}_1 \equiv |\alpha\rangle\langle\beta|$, where $|\alpha\rangle = i\,|1\rangle - 2\,|2\rangle - i\,|3\rangle$ and $|\beta\rangle = i\,|1\rangle + 2|3\rangle$

ii. $\hat{O}_2 \equiv 2\ |1\rangle\langle 1| - i|1\rangle\langle 3| + |2\rangle\langle 3| + i|3\rangle\langle 1| + |3\rangle\langle 2| + |3\rangle\langle 3|$, where {|1⟩ ... is a basis.

(c) The matrix representation of the spin operators along x, y and z directions are gi[ven by]

$$\hat{S}_x = \hbar/2\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \hat{S}_y = \hbar/2\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \text{ and } \hat{S}_z = \hbar/2\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

respectively in the eigenbasis of $\hat{S}_z$.

i. what are the eigenvalues and eigenstates of $\hat{S}_z$ in the eigenbasis of $\hat{S}_z$.

ii. find the eigenvalues and eigenstates of $\hat{S}_x$ and $\hat{S}_y$ in the eigenbasis of $\hat{S}_z$.

iii. if the eigenstates of $\hat{S}_z$ are $|+\rangle$ and $|-\rangle$, state the eigenstates of $\hat{S}_x$ and $\hat{S}$[$_y$ as] combinations of $|+\rangle$ and $|-\rangle$.

iv. state $|+\rangle$ and $|-\rangle$ as column matrixes in the eigenbasis of $\hat{S}_x$.

) The root mean square deviation of an operator $\hat{A}$ in a normalized state $|\psi\rangle$ is defined as

$$\Delta A = \sqrt{\langle(\hat{A} - \langle\hat{A}\rangle)^2\rangle},$$

where $\langle\hat{A}\rangle = \langle\psi|\hat{A}\,|\psi\rangle$ is the expectation value of the operator in the same state.

Show that $\Delta A = \sqrt{\left[\langle\hat{A}^2\rangle - \left(\langle\hat{A}\rangle\right)^2\right]}$, where $\langle\hat{A}^2\rangle$ is the expectation value of $\hat{A}^2$ in the state $|\psi\rangle$.

) The Swartz inequality can be expressed as $\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle \geq |\langle\alpha|\beta\rangle|^2$ for any states $|\alpha\rangle$ and $|\beta\rangle$.

Consider $|\alpha\rangle = (\hat{A} - \langle\hat{A}\rangle)|\psi\rangle$ and $|\beta\rangle = (\hat{B} - \langle\hat{B}\rangle)|\psi\rangle$ and use the identity $|Z|^2 \geq \left[{(Z - Z^*)}/{2i}\right]^2$ for any complex number Z and the Swartz's inequality to show that $\Delta A\,.\,\Delta B \geq {\langle[\hat{A},\hat{B}]\rangle}/{2i}$, where $\langle[\hat{A},\hat{B}]\rangle$ represents the expectation value of the commutator $[\hat{A},\hat{B}]$ in the state $|\psi\rangle$.

) Consider a particle moving in space with position $\vec{r} = (x, y, z)$ and momentum $\vec{p} = (p_x, p_y, p_z)$.

i. find the commutation relation $[\hat{x}, \hat{p}_x]$ and state other similar commutation relations.

ii. show that $\Delta x.\Delta p_x \geq {\hbar}/{2}$ for any particle, where Δx is the uncertainty in measuring the position x and Δp_x is the uncertainty of measuring momentum along the x direction of the particle.

iii. what is the minimum value of $\Delta x.\Delta p_y$?

05. (a) Show that in the non-degenerate stationary perturbation theory, if the Ham can be written as $H = H^{(0)} + \lambda H'$ with $H^{(0)}$ Hermitian, $\lambda <<<$ $H^{(0)}\psi_n{}^{(0)} = E_n{}^{(0)}\psi_n{}^{(0)}$, $n = 1, 2, 3, \ldots\ldots\ldots$, then the first order correction, the energy $E_n{}^{(0)}$ is given by $E_n{}^{(1)} = \langle \psi_n{}^{(0)} | H' | \psi_n{}^{(0)} \rangle$.

(b) A particle of mass *m*, is undergoing a simple harmonic motion in one dim space with frequency ω.

(i) state the Hamiltonian and the particle energy when it is in the n^{th} state derive).

(ii) if a perturbation of the form $H' = \lambda \dfrac{p}{m}$ ($\lambda <<< 1$ and is a constant, momentum) is introduced to the above system, what is the first order c to the n^{th} energy.

(c) Calculate the second order correction, $E_n{}^{(2)}$.

06. (a) (i) Show that the spin orbit interaction energy term for H- atom is

$$H_{so} = \frac{e^2}{16\pi\varepsilon_0 m_e{}^2 c^2} \frac{\vec{L}.\vec{S}}{r^3},$$ where symbols have their usual meanings.

Note that $\left\langle \dfrac{1}{r^3} \right\rangle_{n,l} = \dfrac{1}{n^3 a_0{}^3 \{l(l+1)(2l+1)\}}$.

(ii) Show that $\vec{L}.\vec{S} = \left[\dfrac{(j(j+1) - l(l+1) - s(s+1))}{2} \right] \hbar^2$

(b) Consider a hypothetical atom with a H like nucleus and an orbiting particl negative charge and a mass equal to those of an electron. The spin of th charge is 1.

(i) Draw an energy level diagram showing the splitting of 2p and 1s le spin-orbit coupling. In your diagram, show energy differences fro energy and the degeneracy of each new level. Give the standard spectroscopic notations of the new levels.

(ii) Write down the selection rules associated with j and l quantum numbers and indicate the allowed transitions in the energy level diagram.