EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SECOND EXAMINATION IN SCIENCE - 2009/2010
SECOND SEMESTER (April/May, 2012)
MT 218 - FIELD THEORY
(PROPER & REPEAT)

Answer all Questions | **Time: Two hours**

Q1. State the Gauss's theorem in Electric field.

(a) i. A spherical conductor of radius a is maintained at a constant potential V. It is surrounded by a concentric spherical conducting cell of radius b which is insulated and carries a charge e. Show that at a point at distance r from the center of the sphere $(a \leq r \leq b)$ the potential is given by

$$\frac{1}{4\pi\epsilon_0}\left\{\frac{e}{b} + \left(4\pi\epsilon_0 V - \frac{e}{b}\right)\frac{a}{r}\right\}$$

where ϵ_0 is permittivity of free space.

ii. Assuming that the total charge Z_e of an atomic nucleus is uniformly distributed within a sphere of radius a. Show that the electric potential at a distance r from the center $(r \leq a)$ is

$$V = \left[3 - \left(\frac{r}{a}\right)^2\right]\frac{Z_e}{8\pi\epsilon_0 a}.$$

(b) i. A semi-infinite straight uniformly charged filament has a charge λ per unit length. Find the magnitude and the direction of the field intensity at the point separated from the filament by a distance y and lying on the normal to the filament passing through its end.

ii. Find the electric field $E(r)$, inside and outside of a uniformly charged insulating sphere with total charge Q and radius R.

Q2. If ϕ is the potential due to an axisymmetric system, then we have $\nabla^2\phi = 0$ in free space.

By taking $\phi(r,\theta,\psi) = \frac{1}{r} U(r)\, P(\theta)\, Q(\psi)$, show that

i. $\frac{d^2u}{dt^2} - \frac{du}{dt} - l(l+1) = 0$;

ii. $\frac{d}{dx}\left[(1-x^2)\frac{dp}{dx}\right] + l(l+1)P = 0.$

By solving these two equations show that the solution of the Laplace's equation $\nabla^2\phi = 0$ can be written as

$$\phi = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

where P_l is the solution of the differential equation

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} [(x^2-1)^l].$$

Hint: Laplace equation for spherical co-ordinate system is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\psi^2} = 0.$$

Q3. (a) Using Ampere's circuit law and Biot-Savart law, prove that $\nabla^2\phi = 0$, where ϕ is scalar potential.

(b) Show that the equivalence between Biot-Savart and Ampere's laws will be brought out by determining the magnetic field $\vec{B}$ due to an infinitely long conductor carrying a steady current through it.

(c) A long thin flat strip of metal is of width W and has a current I flowing along it. Find the magnetic induction B at a point P in the plane of the strip at a distance b from the nearest edge.

Q4. (a) State the Newton's law of gravitation.

Derive an equation to find the gravitational force between a uniform sphere of mass M, radius r and a particle of mass m.

(b) State the Gauss's theorem in gravitational field.

State and prove Kepler's first law.

A solid of mass m and radius a with density proportional to the distance from the center is built up a state of infinite dispersion. Show that the loss of potential energy is $\frac{4Gm^2}{7a}$.