EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE - 2009/2010

SECOND SEMESTER (PROPER/REPEAT)

(April 2012)

PH 204 MECHANICS II

Time: 01 hour.

Answer ALL Questions

---

1. Explain the principle underlying the action of a rocket.

   Starting from conservation of linear momentum, show that the general equation of motion for a rocket is

$$F_{\text{ext}} = m\frac{dv}{dt} + u\frac{dm}{dt}$$

   where m is the total mass of the rocket including propellant fuel, F_{ext} is the net external force, u is the effective exhaust velocity relative to the rocket and v is the upward vertical velocity of the rocket relative to stationary earth.

   (a) For vertical motion of a rocket, show that the maximum vertical velocity, V of the rocket at burnout is

$$V = u\left[\ln\left(\frac{m_o}{m_{bo}}\right)\right] - gt_{bo}$$

   where m_o is the initial total mass of rocket and propellant fuel, m_{bo} is the burnout mass, g is the gravitational acceleration and t_{bo} is the final time of burnout.

   (b) Hence, deduce that the mass of the propellant fuel m_p at any instantaneous time t is

$$m_p = m_o\left\{1 - \exp\left[-\left(\frac{V + gt}{u}\right)\right]\right\}.$$

2. (a) A particle of mass m in a central-force field $F(r)$ moves with a constant angular momentum L about the force center. Show that the general equation of the particle's orbit is expressed by

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{L^2u^2}F\left(\frac{1}{u}\right)$$

where r and θ are the plane polar coordinates of the particle and $u = \frac{1}{r}$

(b) A particle of mass m moving in an attractive central-force field is described by $F(r) = -\left(\frac{k}{r^2}\right)$. Show that the orbital motion of the particle is circle of radius $\frac{L^2}{mk}$ when total energy becomes $-\left(\frac{mk^2}{2L^2}\right)$.