EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE 2015/2016

FIRST SEMESTER (May/June, 2018)

AM 306 - PROBABILITY THEORY

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swer all questions | Time : Two hours

lculator and Statistical tables will be provided

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1. (a) State Bayes' theorem.

A new test is developed to identify people who are liable to suffer from some genetic disease in later life. Suppose that 1 in 1000 of the population is a carrier of the disease. Suppose also that the probability that a carrier tests negative is 1%, while the probability that a non carrier tests positive is 5%.

i. A patient has just had a positive test result. What is the probability that the patient is a carrier?

ii. A patient has just had a negative test result. What is the probability that the patient is a carrier?

(b) A random variable X has Poisson distribution with parameter λ. Find the mean, variance of X.

(c) In a certain manufacturing process, 10% of the tools produced turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective, by using

i. the binomial distribution;

ii. the Poisson approximation to the binomial distribution.

2. Define the "moment generating function" of a random variable X.

(a) Show that if X and Y are independent random variables, then $X+Y$ has the mom

generating function

$$M_{X+Y}(t) = M_X(t) + M_Y(t).$$

(b) The probability density function of a Gamma distribution with parameters m and

given by

$$f_X(x) = \begin{cases} \dfrac{\lambda^m x^{m-1} e^{-\lambda x}}{\Gamma(m)} & \text{if} \quad x > 0\,; \\ 0 & \text{otherwise}\,. \end{cases}$$

i. Find the moment generating function of X.

ii. Let X and Y be independent random variables, X having the Gamma distrib with parameters m and λ, and Y having the Gamma distribution with param s and λ. Show that $X+Y$ has the gamma distribution with parameters $m+s$ λ.

(c) Let $X_1, X_2, \cdots, X_n$ be independent random samples of size n from an expone distribution with mean $\frac{1}{\lambda}$.

i. Show that $T = \sum_{i=1}^{n} X_i$ follows the gamma distribution with parameters n and

ii. Hence prove that $2T$ follows the chi-square distribution with $2n$ degrees of free

3. (a) If X and Y are independent exponential random variables with parameter $\lambda > 0$

i. Find the joint probability density function of $U = X + 2Y$ and $V = 2X + Y$

ii. Are X and Y independent?

(b) A random variable X has a gamma distribution with parameters $m = 1$ and Find the probability density function of the random variable e^X.

(c) A random variable X has a Uniform distribution on the interval from 0 to 10. $P\left[X + \frac{10}{X} \geq 7\right]$.

4. (a) Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from the normal distribution with mean μ and variance σ^2.

   i. Find the maximum likelyhood estimators of μ and σ^2.

   ii. Are your estimators of μ and σ^2 unbiased? Justify your answer.

   (b) In measuring reaction time, a psychologist estimates that the standard deviation is 0.05 second. How large a sample of measurements must he take in order to be 95% confident that the error in his estimate of mean reaction time will not exceed 0.01 second?