EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE - 2015/2016

SECOND SEMESTER (Oct./Nov., 2018)

AM 307 - CLASSICAL MECHANICS III

Answer all Questions **Time: Three hours**

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1. (a) Two frames of reference S and S' have a common origin O and S' rotates with a constant angular velocity $\underline{\omega}$ relative to S. If $\underline{r}$ be the position vector of a particle at time t in S', then show that the acceleration of the particle in S is

$$\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \frac{\partial \underline{r}}{\partial t} + \frac{\partial \underline{\omega}}{\partial t} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r}).$$

(b) A projectile is fired from a point on the surface of earth located at colatitude λ with a velocity of magnitude v_0 in a southward direction at an angle α with the horizontal.

i. Find the position of the projectile after time t.

ii. Prove that after time t the projectile is deflected towards the east of the original vertical plane of the motion by the amount

$$\frac{1}{3}\omega g \sin \lambda t^3 - \omega v_0 \cos(\alpha - \lambda) t^2,$$

where ω is the angular velocity of the earth

2. With the usual notations, obtain the equations of motion for a system of N particles in the following form:

(a) $M\underline{f}_G = \sum_{i=1}^{N} \underline{F}_i$;

(b) $\frac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i$, where $\sum_{i=1}^{N} \underline{h}_i = \underline{H}$ and $\underline{h}_i = \underline{r}_i \wedge m_i \underline{v}_i$;

(c) $T = T_G + \frac{1}{2} M \underline{v}_G^2$.

A ladder leaning against a smooth wall makes an angle α with the horizontal when in a position of limiting equilibrium. Show that the coefficient of friction between the ladder and the ground is $\frac{1}{2}\cot\alpha$.

3. (a) With the usual notations obtain the *Euler's* equations of motion for a rigid body, having a point fixed, in the form:

$$A\dot{\omega}_1 - (B-C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = N_2,$$

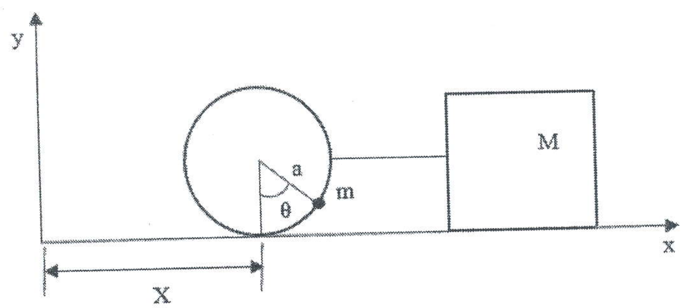
$$C\dot{\omega}_3 - (A-B)\omega_1\omega_2 = N_3.$$

A body moves freely about a point O under no forces. The principle moment of inertia at O being A, $2A$ and $3A$. Initially the angular velocity has components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = \frac{n}{\sqrt{3}}$ about the corresponding principal axes. Show that at time t,

$$\omega_2 = n\tanh\left(\frac{nt}{\sqrt{3}}\right).$$

4. Obtain the Lagrange's equations of motion using D'Alembert's principle for a conservative holonomic dynamical system.

A block of mass M is free to slide on a frictionless horizontal table. The block is rigidly connected to a massless circular hoop of radius a. A particle of mass m is confined to move without friction on the circular hoop, which is vertical(as shown in the following figure)



(a) Write down the Lagrangian of the system.

(b) Apply Lagrange's equations and calculate the frequency of small oscillations about the equilibrium position.

5. With the usual notations, write down the Lagrange's equation for the impulsive motion.

A uniform rod AB of length $2a$ and mass m has a particle of mass M attached to the end B. It is at rest on a smooth horizontal table. If an impulse I is applied at A in a direction perpendicular to AB, and in the plane of the table, then find the initial velocities of A and B.

Prove that the resulting kinetic energy is given by

$$\frac{2I^2(m+3M)}{m(m+4M)}.$$

6. (a) Define Hamiltonian function in terms of Lagrangian function.

Show that, with the usual notations, that the Hamiltonian equations are given by

$$\dot{q_j} = \frac{\partial H}{\partial p_j}, \; \dot{p_j} = -\frac{\partial H}{\partial q_j} \text{ and } \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$

Write down Hamiltonian equations for the following Hamiltonian

$$H = \frac{1}{2}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2 \sin^2\theta}\right).$$

(b) Define the poisson bracket.

i. Show that the Hamiltonian equations of the holonomic system may be written in the form

$$\dot{q}_k = [q_k, H], \quad \dot{p}_k = [p_k, H],$$

and show that for any function $f(q_i, p_i, t)$, $\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H]$.

ii. Show that, if f and g are constants of motion then their poisson bracket $[f, g]$ is also a constant of motion.