EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE - 2013/2014

SECOND SEMESTER (October, 2017)

AM 307 - CLASSICAL MECHANICS

SPECIAL REPEAT

Answer all Questions — Time: Three hours

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1. Two frames of reference S and S' have a common origin O and S' rotates with an angular velocity $\underline{\omega}$ relative to S. If a moving particle P has its position vector as $\underline{r}$ relative to O at time t, show that :

   (a) $\dfrac{d\underline{r}}{dt} = \dfrac{\partial \underline{r}}{\partial t} + \underline{\omega} \wedge \underline{r}$, and

   (b) $\dfrac{d^2\underline{r}}{dt^2} = \dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \dfrac{\partial \underline{r}}{\partial t} + \dfrac{\partial \underline{\omega}}{\partial t} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r})$.

   An object is thrown downward with an initial speed v_0. Prove that after time t the object is deflected east of the vertical by the distance

   $$\omega v_0 \sin \lambda t^2 + \frac{1}{3}\omega g \sin \lambda t^3,$$

   where λ is the earth's co - latitude.

2. (a) With the usual notations, obtain the equations of motion for a system of N particles in the following forms:

   i. $M\underline{f}_G = \sum_{i=1}^{N} \underline{F}_i$,

   ii. $\dfrac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i$,

where $\sum_{i=1}^{N} \underline{h}_i = \underline{H}$ and $\underline{h}_i = \underline{r}_i \wedge m_i \underline{v}_i$.
(State clearly the results that you may use)

(b) The center of a uniform circular disc of radius R and mass M is rigidly mounted on at one end C of a thin light shaft CD of length L. The shaft is normal to the disc at the center. The disc rolls on a rough horizontal plane, the other end D of the shaft being fixed in this plane by a smooth universal joint. If the center of the disc rotates without slipping about the vertical through D with constant angular velocity Ω, find the angular velocity, the kinetic energy and the angular momentum of the disc about D.

3. (a) With the usual notations, obtain Euler's equations of motions for a rigid body having a point fixed, in the following form:

$I_{ox}\dot{\omega}_x - (I_{oy} - I_{oz})\omega_y\omega_z = N_x,$
$I_{oy}\dot{\omega}_y - (I_{oz} - I_{ox})\omega_z\omega_x = N_y,$
$I_{oz}\dot{\omega}_z - (I_{ox} - I_{oy})\omega_x\omega_y = N_z.$

(b) Imagine that a rigid body is rotating about a fixed point with angular velocity $\underline{\omega}$. Further, assume that the coordinate axis coincide with the principal axis. Show that, if T is a kinetic energy and $\underline{N}$ is an external torque acting on the body, then

$$\frac{dT}{dt} = \underline{N}.\underline{\omega}.$$

4. Obtain the Lagrange's equations of motion using D'Alembert's principle for a conservative holonomic dynamical system.

Use the Lagrangian method and obtain the equations of motion for a spherical pendulum of length r.

5. (a) Define Hamiltonian function in terms of Lagrangian function .

Show that, with the usual notations, that the Hamiltonian equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \dot{p}_j = -\frac{\partial H}{\partial q_j} \text{ and } \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$

(b) Write down the Hamiltonian and then find the equation of motion when the particle of mass m is moving on a cartesian coordinate system.

6. (a) Define what is it meant by the poisson bracket.

With the usual notations, show that, for any function $F(p_j, q_j, t)$,

$$\dot{F} = [F, H] + \frac{\partial F}{\partial t},$$

where H is a Hamiltonian.

(b) With the usual notations, prove that:

i. $\frac{\partial}{\partial t}[f, g] = \left[\frac{\partial f}{\partial t}, g\right] + \left[f, \frac{\partial g}{\partial t}\right],$

ii. $[f, p_k] = \frac{\partial f}{\partial q_k}.$

(c) Show that, if f and g are constants of motion then their poisson bracket $[f, g]$ is also a constant of motion.