EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE - 2014/2015

SECOND SEMESTER (Dec., 2017/Jan., 2018)

AM 310 - FLUID MECHANICS

Answer all questions | Time : Two hours

1. (a) Derive the continuity equation for a fluid flow in the form

$$\frac{D\rho}{Dt} + \rho\underline{\nabla}.\underline{q} = 0,$$

where ρ and $\underline{q}$ are the density and the velocity of the fluid.

Hence, establish the equation of continuity for an incompressible fluid in the form $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ in cartesian coordinates, where u, v and w are the cartesian components of the velocity.

(b) If the velocity of an incompressible fluid at any point (x, y, z) is given by $r^{-5}\left(3xz, 3yz, 3z^2 - r^2\right)$, where $r^2 = x^2 + y^2 + z^2$, then prove that the fluid motion is possible and the velocity potential is z/r^3.

Also determine the streamlines.

2. An infinite inviscid fluid of constant density is subjected to a force $\mu r^{-7/3}$ per unit mass directed towards the origin O, where μ is a constant and r is a distance from O. Initially the liquid is at rest, and there is a cavity bounded by a sphere $r = a$. The pressure in the cavity as well as at infinity is zero. If the radius of the cavity at time t is $R(t)$, show that $R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{3}{4}\mu R^{-4/3}$, and that the cavity will be filled after a time $\frac{\pi a^{5/3}}{\sqrt{10\mu}}$.

3. (a) Let a two-dimensional source of strength m be situated at origin. Show that the complex potential w at a point $P(z)$ due to this source is given by $w = -m \ln z$.

(b) In the region bounded by a fixed quadrantal arc and its radii, there is a two-dimensional fluid motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the streamline passing through the point (r, θ) in polar coordinates and leaving either end at an angle α with the radius is

$$r^2 sin(\alpha + \theta) = a^2 \sin(\alpha - \theta),$$

where a is the radius of quadrantal arc.

4. Write down the Bernoulli's equation for steady motion of an inviscid incompressible fluid.

A three-dimensional doublet of strength μ whose axis is in the direction of $\overrightarrow{Ox}$ is distant a from a rigid plane $x = 0$ which is the sole boundary of liquid of density ρ, infinite in extent. If the pressure at infinity is Π, find the pressure at a point on the boundary distant r from the doublet.

Determine the point on the plane where the pressure is least.