EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2014/2015

SECOND SEMESTER (Oct., 2017)

AM 310 - FLUID MECHANICS

(Special Repeat)

Answer all questions **Time : Two hours**

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1. (a) Let the velocity of a fluid flow be $\underline{V} = (1 + At)\underline{i} + x\underline{j}$, where A is a constant.

   i. Find the equation of streamline passing through the point (x_0, y_0) at $t = t_0$.

   ii. Obtain the equation of path line of a fluid element which comes to (x_0, y_0) at $t = t_0$.

   Hence, show that the streamline and path line coincide when $A = 0$.

   (b) An inviscid fluid moves through a straight long tube of narrow core. The density of the fluid at a distance x from one end of the tube at time t is $\rho_0 e^{a(x-ct)}$, where a, c and ρ_0 are constants. If the velocity of the fluid at the end, $x = 0$, is equal u then show that the velocity of the fluid at a distance l from that end is $c + (u - c)e^{-al}$.

2. (a) Let a gas occupy the region $r \leq R$, where R is a function of time t, and a liquid of constant density ρ lie outside the gas. If the velocity at $r = R$, the gas liquid boundary, is continuous then show that the pressure p at a point $P(\underline{r}, t)$ in the liquid is given by

$$p(r) = \Pi + \rho \left[\frac{1}{r} \frac{d}{dt}(R^2 \dot{R}) - \frac{1}{2} \left(\frac{R^2 \dot{R}}{r^2} \right)^2 \right],$$

   where Π is the pressure at infinity.

   Show also that the gas liquid interface pressure is given by

$$p(R) = \Pi + \frac{\rho}{2R^2} \frac{d}{dR}(R^3 \dot{R}^2).$$

(b) If the gas obeys the Boyle's law $pv^{\frac{4}{3}} =$ constant, where v is the volume of the gas, and expands from rest ar $R = a$ to a position of rest at $R = 2a$, show that the ratio of initial pressure of the gas to the pressure of the liquid at infinity is 14:3.

3. (a) Let a two-dimensional source of strength m be situated at origin. Show that the complex potential w at a point $P(z)$ due to this source is given by $w = -m \ln z$.

   (b) In the part of an infinite plane bounded by a circular quadrant AB and the radii OA, OB; there is a two-dimensional motion due to a source of strength m at A and a sink of strength m at B. Find the velocity potential of the motion at a point $P(r, \theta)$.

   Show that the fluid which issues from A in the direction making an angle α with OA follows the path whose polar equation is

$$r = a\sin^{\frac{1}{2}} 2\theta \left[\cot\alpha + \sqrt{\cot^2\alpha + \operatorname{cosec}^2 2\theta}\right]^{\frac{1}{2}},$$

   where $OA = OB = a$.

4. Write down the Bernoulli's equation for steady motion of an inviscid incompressible fluid.

   Let a fluid of density ρ fill the region of space on the positive side of the x axis with the plane determined by the y axis and z axis being a fixed boundary. If a two dimensional source of strength m is situated at point $(a, 0)$, find the points on the boundary at which the velocity is maximum. Show that the resultant thrust on the area formed by the part of the axis of y which lies between $y = \pm b$ and unit length along the z axis is

$$2\Pi b - 2m^2\rho\left[\frac{1}{a}\tan^{-1}\left(\frac{b}{a}\right) - \frac{b}{a^2 + b^2}\right],$$

   where Π is the pressure at infinity.