EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE-2010/2011

FIRST SEMESTER (April, 2013)

MT304 - GENERAL TOPOLOGY

Answer all questions

Time : Two hours

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1. Define the following terms:

   - Topology on a set;
   - Interior of a set.

   (a) Let X be a non-empty set. Let τ be the collection of subsets of X containing the empty set Φ and all subsets whose complements are finite. Is (X, τ) a topological space? Justify your answer.

   (b) Let A be a non-empty subset of a topological space (X, τ). Prove that

   i. the interior of A is the largest open set contained in A.

   ii. A is open if and only if $A = A^{\circ}$.

   (c) Let $X = \{1, 2, 3\}$ and $\tau = \{X, \Phi, \{1, 2\}, \{2, 3\}, \{2\}\}$. Let $A = \{1, 2\}$. Find the interior of A.

2. (a) If (X, τ) is a topological space, where $\tau = \{A \subseteq X \mid A = \Phi \text{ or } A^c \text{ is finite}\}$ and X is an infinite set. Prove that $\overline{A} = X$ for any infinite subset A of X.

   (b) Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that $A \subseteq Y$ is a closed subset of Y in (Y, τ_Y) if and only if $A = F \cap Y$ for some closed subset F of X in (X, τ).

   (c) Let f be a function from a topological space (X, τ_1) into a topological space (Y, τ_2).

      i. Prove that, f is continuous on X if and only if $f^{-1}(G)$ is open in X for every open subset G in Y.

      ii. Prove that, f is continuous on X if and only if $f^{-1}(A^\circ) \subseteq \{f^{-1}(A)\}^\circ$ for every subset A of Y.

3. Let (X, τ) be a topological space. Prove that the following statements are equivalent:

   (i) X is connected;

   (ii) X cannot be expressed as the union of two disjoint non-empty closed sets;

   (iii) The only subsets of X which are both open and closed are X and Φ;

   (iv) The set of all frontier points of A, denoted by Fr A, is non-empty, for any non-empty proper subset A of X;

   (v) There is no continuous function from X onto Y, when $Y = \{0, 1\}$ has the discrete topology.

4. Define the following terms:

   - Frechet space (T_1);
   - Housdorff space (T_2);
   - Compact set.

   (a) Prove that a closed subset of a compact topological space is compact.

   (b) Prove that every compact subset of a Housdorff topological space is closed.

   (c) Prove that every Housdorff space is a Frechet space. Is the converse true? Justify your answer.