# EASTERN UNIVERSITY, SRI LANKA

## DEPARTMENT OF MATHEMATICS

## THIRD YEAR SPECIAL REPEAT EXAMINATION IN SCIENCE -2010/2011

## (JUNE/JULY, 2014)

## MT 305 – OPERATIONAL RESEARCH

Answer **all** questions. Time: **Three** hours.

01. A company manufactures two products A and B. Each requires the use of two assembly lines $L_1$ and $L_2$. One unite of A requires 12 hours from $L_1$ and 4 hours from $L_2$. One unit of B requires 4 hours from $L_1$ and 8 hours from $L_2$. During a certain period, the maximum number of hours available in $L_1$ and $L_2$ lines are respectively 60 hours and 40 hours. Profits per unit of A and B are Rs. 90 and Rs. 70, respectively.

(a) By defining variables clearly, formulate a linear programming model to determine the optimal product mix that maximizes the profit.

(b) Use the graphical method to find the optimal solution for the above linear programming model.

(c) Write down the maximum profit.

02. A hospital dietician must prepare breakfast menus every morning for the hospital patients. Dietician's responsibility is to make sure that minimum daily requirements for vitamins A and B are met. At the same time, the menus must be kept at the lowest possible cost to avoid waste. The main breakfast staples providing vitamins A and B are eggs, bacon and cereal. The vitamins requirements and vitamins contribution for each staple are in following table.

| Vitamins | Vitamin contribution | | | Minimum daily requirement(mg) |
|---|---|---|---|---|
| | mg/Egg | Mg/Bacon | Mg/cereal cup | |
| A | 2 | 4 | 1 | 16 |
| B | 3 | 2 | 1 | 12 |

(P. T. O.)

An egg, a bacon strip and a cup of cereal will cost Rs. 4.00, Rs. 7.00 and Rs. 2.00 respectively. The dietician wants to know how much of each staple to serve in order to meet the minimum daily requirements while minimizing total cost.

(a) Formulate a linear programming model for this problem.

(b) Use the Simplex method to find the optimal solution of the above linear programming model.

(c) What is the minimum cost of breakfast per person?

03. Using Revised Simplex method, solve the following linear programming model:

Maximize

$$Z = 2X_1 + 3X_2 - X_3 + 4X_4,$$

subject to the constraints:

$$X_1 - 2X_2 + X_4 \leq 10,$$
$$X_1 + X_2 + 2X_3 \leq 16,$$
$$(1/2)\, X_2 - X_3 - X_4 \leq 8,$$

where $X_1, X_2, X_3, X_4 \geq 0$.

04. A manager has to assign four different workmen to four different jobs. The times taken by each workman to complete each job differ as shown in the table below.

| **Workman** | **Time (hours) requires for each job** | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| **$W_1$** | 10 | 30 | 19 | 15 |
| **$W_2$** | 20 | 19 | 24 | 25 |
| **$W_3$** | 18 | 17 | 20 | 9 |
| **$W_4$** | 14 | 12 | 1 | 25 |

(a) Formulate a mathematical model for this assignment problem. Clearly define the variables and state the constraints.

(b) How should the jobs be assigned to workman to minimize the total man-hours? Write down the minimum total man-hours need to complete all jobs.

05. A transporting company plans to transport some logs from three harvesting sites $S_1$, $S_2$ and $S_3$ to three sawmills $M_1$, $M_2$ and $M_3$ at the minimum cost. The distance from each site to each sawmill, number of truckloads of logs available at each site and number of truckloads of logs each sawmill demands, are given in following table. The average cost of transportation is \$2 per kilometer for both loaded and empty trucks.

| **Logging sites** | **Distance to mills(in km)** | | | **Maximum truckloads from logging site per day** |
|---|---|---|---|---|
| | **$M_1$** | **$M_2$** | **$M_3$** | |
| $S_1$ | 8 | 15 | 50 | 20 |
| $S_2$ | 10 | 17 | 20 | 30 |
| $S_3$ | 30 | 26 | 15 | 45 |
| **Mill demand (Truckload per day)** | 30 | 35 | 30 | - |

(a) Defining variables clearly, build up the mathematical model for the above transportation problem.

(b) Find the initial feasible solution by using Row minima method.

(c)Check the optimality of the solutions obtained in part (b) by using modified distribution (MODI) method.

(d) Find the minimum total cost.

06. Consider the road network as in the following figure, where distances (in km) between adjacent cities are summarized. Find the shortest route from city 1 to city 10, by using Systematic method.