EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE – 2009/2010

SECOND SEMESTER (SPECIAL REPEAT)

FEBRUARY/MARCH 2013

PH 305 FUNDAMENTALS OF STATISTICAL PHYSICS

Time: 01 hour

Answer ALL Questions

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1. Explain what is meant by the "single-particle partition function" Z of a system of N localized independent particles whose indivisible energy states are known.

   Derive the relation between the thermal average energy and the single particle partition function for a system of N weakly interacting distinguishable particles.

   A system of N weakly interacting identical particles is in thermal equilibrium with a large reservoir at absolute temperature T. Each particle can take energies ε_1 and ε_2.

   i. Write down an expression for the partition function for a single particle.

   ii. What is the average thermal energy of a single particle?

   iii. Obtain an expression for the heat capacity at constant volume of the system.

2. Explain the terms "microstate", and "density of states" as used in statistical physics?

   State the conditions for a system to obey Maxwell-Boltzmann (M-B) statistics and derive an expression for the M-B distribution function in terms of the partition function of the system.

   An ideal gas composed of monatomic molecules can be described by M-B statistics. Given that the number of molecules of an ideal gas within the energy range ε and $\varepsilon + d\varepsilon$ is given by

$$g(\varepsilon)d\varepsilon = \frac{2\pi V(2m)^{\frac{3}{2}}\varepsilon^{\frac{1}{2}}}{h^3}d\varepsilon$$

(a) Show that the partition function of the ideal gas is given by

$$Z = \frac{V(2\pi mkT)^{\frac{3}{2}}}{h^3}$$

where the symbols have their usual meanings.

(b) Prove that the most probable velocity v_{mp} of the molecules of an ideal gas is given by the relation

$$v_{mp}^2 = \frac{2}{3} v_{rms}^2$$

where v_{rms} is the root mean square velocity of the molecules. Sketch a typical plot for most probable energy distribution for two different temperatures.

You may find the following integrals useful.

$$\int_0^{\infty} x^{\frac{1}{2}} e^{-x}\, dx = \frac{\sqrt{x}}{2}$$

and

$$\int_0^{\infty} v^3 e^{-\frac{mv^2}{2kT}}\, dv = \frac{1}{2}\left(\frac{2kT}{m}\right)^2.$$