EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

[EX]TERNAL DEGREE EXAMINATION IN SCIENCE 2008/2009

SECOND YEAR SECOND SEMESTER (Mar./May, 2016)

EXTMT 202 - METRIC SPACE (REPEAT)

[Answ]er All Questions | Time : Two Hours

[1.] Define the terms "metric space" and "diameter" of a metric space.

(a) Let l^p be the set of all sequences of real numbers (x_n) for which $\sum_{i=1}^{\infty} | x_n |^p$ is convergent where $1 < p < \infty$. Show that the function $d : l^p \times l^p \longrightarrow \mathbb{R}$, defined by

$$d(x,y) = \left(\sum_{i=1}^{\infty} | x_i - y_i |^p \right)^{1/p} \qquad \forall x = (x_n), \quad y = (y_n) \in l^p,$$

is a metric on l^p.

(b) Let (X, d) be a metric space. Show that a real valued function d_1 on $X \times X$ defined by

$$d_1(x,y) = \frac{d(x,y)}{1 + d(x,y)} \qquad \forall x, y \in X,$$

is also a metric on X. Find the diameter of X with respect to the metric d_1.

2. Let A be a subset of a metric space (X, d). Define the following terms:

- interior point of A;
- limit point of A;
- interior of A.

(a) Prove that the interior of a subset of a metric space is the largest open s in A.

(b) Prove that a subset A of a metric space is closed if and only if A con limit points of A.

(c) Prove that if A is a subset of a metric space (X, d) then $X \setminus A^\circ =$ $X \setminus \overline{A} = (X \setminus A)^\circ$.

3. (a) Prove that two open sets are separated if and only if they are disjoint.

(b) Prove that a metric space X is disconnected if and only if there exists a proper subset of X which is both open and closed.

(c) Let (X, d) be a compact metric space. Prove that if A is a closed subset A is compact.

(d) Prove that every compact subset of a metric space is bounded. Is the c this result true? Justify your answer.

4. What is meant by a function from a metric space (X, d) to a metric space continuous at a point $a \in X$.

(a) Let f be a function from a metric space X into a metric space Y. Prove continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

(b) Let f be a function from a metric space X into a metric space Y. Prove is continuous on X and A is a compact subset of X then $f(A)$ is compac

(c) Let f be a function from a metric space X into a metric space Y. Prove is continuous on X and A is a connected subset subset of X then $f(A)$ is subset of Y.