EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
XTERNAL DEGREE EXAMINATION IN SCIENCE -2008/2009
SECOND YEAR, SECOND SEMESTER (Jun/Sept, 2015)
XTMT 203-ALGEBRA-II(EIGEN SPACE AND QUADRATIC FORMS)

nswer all Questions — Time: Two hours

1. Define the terms *eigenvalue* and *eigenvector* of a linear transformation. [10 marks]

   (a) (i) Let V be a vector space. Prove that eigen vectors that corresponding to distinct eigen values of a linear transformation $T : V \to V$ are linearly independent. [30 marks]

   (ii) If A is an $n \times n$ real matrix and λ is an eigen value of the real symmetric matrix $(I_n + A^T A)$, then show that $\lambda \geq 1$, where I_n is the $n \times n$ identity matrix. [20 marks]

   (b) Let

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix}.$$

   Find a non-singular matrix P such that $P^{-1}AP$ is diagonal. [40 marks]

2. (a) Define the term *positive definite* matrix. [10 marks]

   (b) Prove the followings:

   i. a real symmetric $n \times n$ matrix A is positive definite if and only if all the eigen values of A are positive; [30 marks]

ii. the eigen values of a real symmetric Hermition matrix are real.

[20 mark

(c) Let

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Find the orthogonal matrix P such that $P^T AP$ is diagonal. [40 mark

3. (a) State the *Cayley Hamilton* theorem. [20 mark

Find the minimum polynomial of the square matrix

$$\begin{pmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix}.$$

[40 mark

(b) Find an orthogonal transformation which reduces the following quadratic for to a diagonal form

$$x_1^2 + 2x_2^2 + 3x_3^2 - 4x_1x_2 - 4x_2x_3.$$

[40 mark

4. (a) Let λ_1 and λ_2 be two distinct roots of the equation $|A - \lambda B| = 0$, where A ar B are real symmetric matrices, and let u_1 and u_2 be two vectors satisfying t following

$$(A - \lambda_i B)u_i = 0 \quad \text{for } i = 1, 2.$$

Prove that $u_1^T B u_2 = 0$. [30 mar

(b) Simultaneously diagonalize the following quadratic forms

$$\phi_1 = x_1^2 + 2x_2^2 + 8x_2x_3 + 12x_1x_2 + 12x_1x_3,$$

$$\phi_2 = 3x_1^2 + 2x_2^2 + 5x_3^2 + 2x_2x_3 - 2x_1x_3.$$

[70 mar