EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

ERNAL DEGREE EXAMINATION IN SCIENCE - 2008/2009

ECOND YEAR FIRST SEMESTER (July/August, 2015)

EXTMT 207 - NUMERICAL ANALYSIS

(REPEAT)

er all Questions Time: Two hours

(a) Define what is meant by:

i. *absolute error*;

ii. *relative error* .

Let $p = 0.54617$ and $q = 0.54601$. Use four-digit arithmetic to approximate $p - q$, and determine the absolute and relative errors when rounding and chopping.

(b) i. Show that the polynomial nesting technique can be used to evaluate

$$f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99.$$

ii. Use three - digit rounding arithmetic and the formula given in the statement of part (a) to evaluate $f(1.53)$. Evaluate the absolute error and relative error.

iii. Repeat the calculation in part(b) using the nesting form of $f(x)$ that was found in part (a) . Compare the approximations with part (b).

2. (a) Let $x = \phi(x)$ be the rearrangement of the equation $f(x) = 0$ and de

iteration,

$$x_{n+1} = \phi(x_n), \qquad n = 0, 1, ...$$

with the initial value x_0. If $\phi'(x)$ exists and is continuous such that $|\phi$

$K < 1$ for all x, then show that the sequence x_n generated by the above i

converges to the unique root α of the equation $f(x) = 0$. Find a real roo

equation

$$f(x) = x^3 + x^2 - 1 = 0$$

by the method of iteration.

(b) Given an initial guess x_0, derive Newton-Rapshon method to find a bet

proximation x_1 for approximating the root of a function $f(x)$.

The equation

$$x^2 - 1 - \sin x = 0$$

has roots near 0.6 and 1.6. Use the Newton-Rapshon method to find thes

subject to a tolerance of $\varepsilon = 10^{-6}$.

3. (a) Suppose that $x_0, x_1, \ldots, x_n$ are distinct numbers in the interval $[a,$

$f \in C^{n+1}[a, b]$. Obtain a unique polynomial $P_n(x)$ of degree at most n

property

$$f(x_k) = P_n(x_k) \qquad \text{for each } k = 0, 1, 2, \ldots, n$$

and show that

$$f(x) - P_n(x) = (x - x_0)(x - x_1)\ldots(x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!},$$

where $\xi \in [a, b]$.

(b) i. Use Lagrange's method to find the interpolating polynomial for the da

i	0	1	2	3
x_i	1	2	3	4
$\ln x_i$	0	0.693	1.099	1.386

ii. Approximate $\ln(2.718)$ using the polynomial obtained in part (i).

iii. Find an upper bound on the error for the Lagrange interpolating polynomial on the interval $[1, 4]$.

(a) Use the Jacobi method to approximate the solution of the following system of linear equations.

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 + 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3 \end{aligned}$$

Continue the iterations until two successive approximations are identical when rounded to three significant digits.

(b) With the usual notations, the Simpson's rule is given by

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = \frac{h}{3}\left(f_{i-1} + 4f_i + f_{i+1}\right) - \frac{1}{90}h^5 f^{(iv)}(\xi_i), \text{ where } \xi_i \in [x_{i-1}, x_{i+1}].$$

Obtain the composite Simpson's rule, and show that the composite error is less than or equal to

$$\frac{1}{180}h^4(b-a)\left|f^{(iv)}(\xi)\right|, \text{ where } \left|f^{(iv)}(\xi)\right| = \max_{a \le x \le b}\left|f^{(iv)}(x)\right|.$$

Hence show that composite Simpson's rule is exact for all polynomials of degree 3 or less.