EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2009/2010
SECOND YEAR, FIRST SEMESTER (June/Sept., 2012)
EXTMT 207 - NUMERICAL ANALYSIS
(PROPER & REPEAT)

Answer all questions — Time: Two hours

Q1. (a) Write the suitable form of any non-zero number $x \in F$, where F represents the set of all floating point numbers, and identify the terms involved.

(b) Define the relative round-off error, and explain with an illustrative example.

(c) Find the absolute and relative errors if the computed answer of the exact value 10.147 is 10.159.

(d) A function $f(x) = x^3 - 3x^2 + 3x - 1$ is rearranged in a nested form given by

$$g(x) = [(x-3)x+3]x - 1.$$

Find $f(2.19)$ and $g(2.19)$ using 3-digit rounding. If the true value of $f(x)$ and $g(x)$ at $x = 2.19$ is 1.685159, compare the errors, and state the significance of this problem.

Q2. (a) (i) Let $x = g(x)$ be an arrangement of an equation $f(x) = 0$, which has a root α in the interval I. Suppose that $g'(x)$ exists and is continuous in I such that

$$|g'(x)| \leq h < 1, \ \forall x \in I,$$

where $0 < h < 1$.

Prove that for any given x_0, the sequence $\{x_r\}$, $r = 0, 1, 2, \ldots$, defined by

$$x_{r+1} = g(x_r)$$

converges to the root α, and such α is unique.

(ii) Following iterative formulas are proposed to find a real root of the equation $f(x) = x^3 + x^2 - 1 = 0$, using the iterative method given in (i).

$$x_{r+1} = \frac{1}{\sqrt{x_r + 1}}. \quad (1)$$

$$x_{r+1} = \frac{1}{x_r^2} - 1. \quad (2)$$

Check the applicability of iterative equations (1) and (2) in finding the real root of $f(x)$.

(b) Derive the Newton-Raphson method using Taylor series or otherwise. Carry out four iterations to find x, correct to 4-decimal points, such that

$$f(x) = x^4 - 5 = 0$$

with an initial estimate $x_0 = 2$.

Q3. (a) Write down the divided difference table for e^x using the values

x	e^x
0.0	1.00000
0.4	1.49182
0.9	2.45960
1.5	4.48169
1.8	6.04965.

Estimate $e^{1.2}$, correct to 4-decimal places, using second and third degree interpolation polynomials. If the exact value of $e^{1.2}$ is 3.3201, which interpolation polynomial gives the better estimate? Justify your answer.

(b) Use the Composite Trapezium rule with 2, 4 and 8 sub-intervals to estimate the integral

$$I = \int_1^2 e^x \, dx.$$

If the exact value of I is 4.67078, tabulate the error in each case. What can you say about the accuracy with respect to step size?

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Q4. (a) Solve the system of equations

$$\begin{aligned} 4x_1 + 4x_2 + x_3 + 4x_4 &= 12 \\ 2x_1 + 5x_2 + 7x_3 + 4x_4 &= 1 \\ 10x_1 + 5x_2 - 5x_3 &= 25 \\ -2x_1 - 2x_2 + x_3 - 3x_4 &= -10 \end{aligned}$$

using the Gaussian elimination.

(b) Solve the system of equations

$$\begin{aligned} 16x_1 - 4x_2 + 4x_3 &= 24 \\ -4x_1 + 5x_2 + 3x_3 &= -6 \\ 4x_1 + 3x_2 + 14x_3 &= 15 \end{aligned}$$

by applying the Jacobi iteration (complete 3 iterations with rounding correct to 4-decimal points) using the initial guess $x_1^{(0)} = 0, x_2^{(0)} = 0$ and $x_3^{(0)} = 0$.