EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2008/2009
THIRD YEAR SECOND SEMESTER (April/May - 2016)
EXTMT 307 - CLASSICAL MECHANICS
REPEAT

Answer all Questions **Time: Three hours**

1. Two frames of reference S and S' have a common origin O and S' rotates with an constant angular velocity $\underline{\omega}$ relative to S. If a moving particle P has its position vector as $\underline{r}$ relative to O at time t, show that :

   (a) $\dfrac{d\underline{r}}{dt} = \dfrac{\partial \underline{r}}{\partial t} + \underline{\omega} \wedge \underline{r}$, and

   (b) $\dfrac{d^2\underline{r}}{dt^2} = \dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \dfrac{\partial \underline{r}}{\partial t} + \dfrac{\partial \underline{\omega}}{\partial t} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r})$.

   An object is thrown downward with an initial speed v_0. Prove that after time t the object is deflected east of the vertical by the amount

$$\omega v_0 \sin\lambda t^2 + \frac{1}{3}\omega g \sin\lambda t^3,$$

   where λ is the earth's co - latitude.

2. (a) With the usual notations, obtain the equations of motion for a syster particles in the following forms:

i. $M\underline{f}_G = \sum_{i=1}^{N} \underline{F}_i$,

ii. $\frac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i$,

where $\sum_{i=1}^{N} \underline{h}_i = \underline{H}$ and $\underline{h}_i = \underline{r}_i \wedge m_i \underline{v}_i$.

(State clearly the results that you may use)

(b) A solid of mass M is in the form of a tetrahedron $OXYZ$, the edges OX,(of which are mutually perpendicular, rests with XOY on a fixed smoo izontal plane and YOZ against a smooth vertical wall. The normal rough face XYZ is in the direction of a unit vector $\underline{n}$. A heavy uniform of mass m and center C rolls down the face causing the tetrahedron to a velocity $-V\underline{j}$ where $\underline{j}$ is the unit vector along OY
If $\overrightarrow{OC} = \underline{r}$, then prove that

$$(M+m)V - m\dot{\underline{r}} \cdot \underline{j} = \text{constant}$$

and that

$$\frac{7}{5}\,\ddot{\underline{r}} = \underline{f} - \underline{n}(\underline{n} \cdot \underline{f})\ ,$$

where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and $\underline{g}$ is the acceleration of gravity.

3. With the usual notation obtain the Euler's equations for the motion of th body, having a point fixed, in the form:

$$A\dot{\omega}_1 - (B-C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = N_2,$$

$$C\dot{\omega}_3 - (A-B)\omega_1\omega_2 = N_3.$$

A body moves about a point O under no forces. The principle moment of at O being $3A$, $5A$ and $6A$. Initially the angular velocity has components ω $\omega_2 = 0$, $\omega_3 = 3$ about the corresponding principal axes. Show that at time t,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tan\left(\frac{nt}{\sqrt{5}}\right).$$

4. Obtain the Lagrange's equations of motion using D'Alembert's principle for a conservative holonomic dynamical system.

Use the Lagrangian method and obtain the equations of motion for a spherical pendulum of length r.

5. (a) Define *Hamiltonian* function in terms of Lagrangian function .

Show that, with the usual notations, that the Hamiltonian equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \; \dot{p}_j = -\frac{\partial H}{\partial q_j} \; \text{and} \; \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$

(b) Prove that if the time t does not occur in Lagrangian function L, then the Hamiltonian function H is also not involved in t.

(c) Write down the Hamiltonian function H and then find the equation of motion for a simple pendulum.

6. (a) Define the poisson bracket.

Show that for any function $f(q_i, \; p_i, \; t)$,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, \; H],$$

where H is a Hamiltonian function.

(b) With the usual notations, prove that:

i. $\frac{\partial}{\partial t}[f, \; g] = \left[\frac{\partial f}{\partial t}, \; g\right] + \left[f, \; \frac{\partial g}{\partial t}\right];$

ii. $[f, \; q_k] = -\frac{\partial f}{\partial p_k};$

iii. $[f, \; p_k] = \frac{\partial f}{\partial q_k}.$

(c) Show that, if F and g are constant of motion then their poisson bracket $[f, \; g]$ is a constant of motion.