

## CONSTRUCTION OF HARTSHORN BRIDGE AND AC SUSCEPTIBILITY MEASUREMENTS ON PARA AND DIAMAGNETIC MATERIALS

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### ABSTRACT

The ac mutual inductance technique is one method to study the magnetic properties of materials, which involves measuring the ac magnetic susceptibility of a sample using Hartshorn bridge. This bridge measures the in-phase and the out-of-phase components of the complex ac susceptibility  $\chi'$  and  $\chi''$  of a sample where  $\chi'$  and  $\chi''$  show magnetic susceptibility and the change in energy of the sample respectively.

In this work, a Hartshorn bridge was designed and constructed and the measurements of  $\chi'$  and  $\chi''$  were carried out on the para and the diamagnetic samples at the frequency of 14kHz and the ac field of 0.02 Gauss while the sample was moved into the secondary coil. The  $\chi'$  of para and diamagnetic samples showed positive and negative magnetic susceptibilities respectively in accordance with their para and diamagnetic behaviour. The  $\chi''$  exhibited changes in energy of the samples during their magnetic transitions. The measurements of  $\chi'$  were also performed on the paramagnetic sample when the temperature of the sample decreased and it was found that  $\chi'$  increased with decreasing temperature which is in good agreement with Curie's law for paramagnetic material.

Although the measurements of  $\chi'$  and  $\chi''$  are reported at low temperatures mainly on superconductors, our work suggests that these measurements could also be carried out on magnetic materials at and above room temperature.

**Key Words:** ac magnetic susceptibility, Hartshorn bridge, Paramagnetic materials, Diamagnetic materials

## INTRODUCTION

There are many experimental techniques to characterize the magnetic materials, among which ac inductance technique is most convenient method.

The mutual inductance technique [1] using Hartshorn bridge [2] allow one to monitor the temperature and magnetic field dependence of the sample's ac magnetic susceptibility which is a complex quantity usually denoted by,

$$\chi = \chi' + i\chi''$$

Where  $\chi'$  and  $\chi''$  are the real part (in-phase component) and the imaginary part (out-of phase component) of the complex ac susceptibility.

In general the temperature dependence of the ac susceptibility associated with a small ac magnetic field reflects the magnetic transition. It is shown that with increasing temperature the in-phase component  $\chi'$  of the fundamental susceptibility exhibits para and diamagnetic transition, while the out-of-phase component  $\chi''$  of the fundamental susceptibility shows energy loss or increase inside the magnetic material during the magnetic transition.

## Experimental Techniques

### Hartshorn Bridge

There are several techniques that can be used to measure magnetic susceptibility, one of the more popular is an adoption of the ac Hartshorn bridge in which the specimen is placed in the core of a pick-up coil, and the variation of susceptibility with temperature is measured as a change in the mutual inductance of the coil system.

Hartshorn ac bridge would measure the change of mutual inductance produced by moving a specimen inside a coil system. The measuring (pick-up) coil contains two identical secondaries,

which, are connected in opposition and a coaxial primary. In principal, the net voltage induced across the two secondaries is zero until a specimen is inserted into one of them. When the specimen is inserted into one of the secondaries the mutual inductance would be induced in the coil system due to the susceptibility of the specimen [3]. This inductance would be measured by a lock-in amplifier [4].

### Susceptibility of a Specimen inside the Secondary Coil

The basic Hartshorn bridge circuit consists of a simple mutual inductance arrangement which consists of a two identical secondaries and a coaxial primary. The secondaries are separated along the axis of the primary coil and are connected in opposition. Let  $M_0$  be the mutual inductance when absence of the specimen in the coil and  $M$  is the mutual inductance of the coil at presence of specimen of volume  $v$ .

Then the mutual inductance of the system  $M$  is represented as

$$M = M_0 (1 + f\chi)$$

Where  $f = v/V$  is called filling factor which is positive,  $V$  is the volume of one of the secondaries and  $\chi$  is the susceptibility of

the specimen [5]. For paramagnetic specimen  $\chi > 0$ ,  $\frac{M - M_0}{M_0 f} > 0$  and  $M - M_0 > 0$  and for diamagnetic specimen  $\chi < 0$  and  $M - M_0 < 0$ .

### Construction of Hartshorn Bridge

#### Construction of Coil

The Hartshorn bridge contains a measuring coil (pick-up coil) of two identical secondaries and a coaxial primary. We constructed these coils using the locally made coil winding machine.

Each of two secondaries has a length of 2cm and consists of 3,000 turns while the primary coil consists of 10,000 turns and has a length of 10cm. The primary

and secondaries were wound using 42SWG ( diameter of 0.102mm) enameled, copper wires. Further the resistance of the primary and secondary coils were measured to be 2.5k $\Omega$  and 1.15k $\Omega$  respectively at the room temperature. The dimensions of the coils are presented in figure. 1

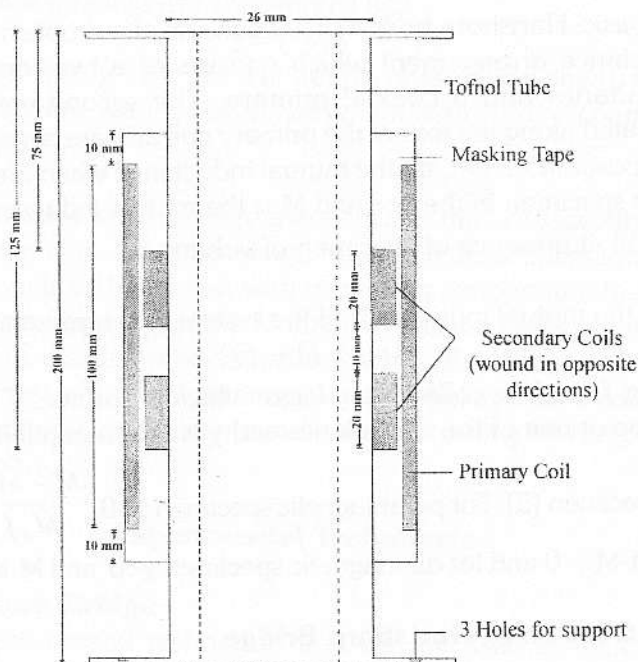


Fig. 1 Schematic diagram and dimensions of the measuring coil

The primary and secondaries were wound on a Tofnol tube of dimensions 20cm in long and 5cm in diameter. The two secondaries were wound oppositely as two identical halves.

## Electronic Circuit

An elaborated electronic circuit has been designed in this experiment to measure the magnetic susceptibility  $\chi'$  and  $\chi''$  of magnetic materials. It was constructed using locally available resources.

The major parts of the electronic circuit of the a.c Hartshorn bridge are primary coil P, two oppositely wound secondaries S, mutual inductance box M, Helipot H, sine-wave generator A, and two phase lock-in amplifier L. The diagram of the electronic circuit is shown in fig. 2.

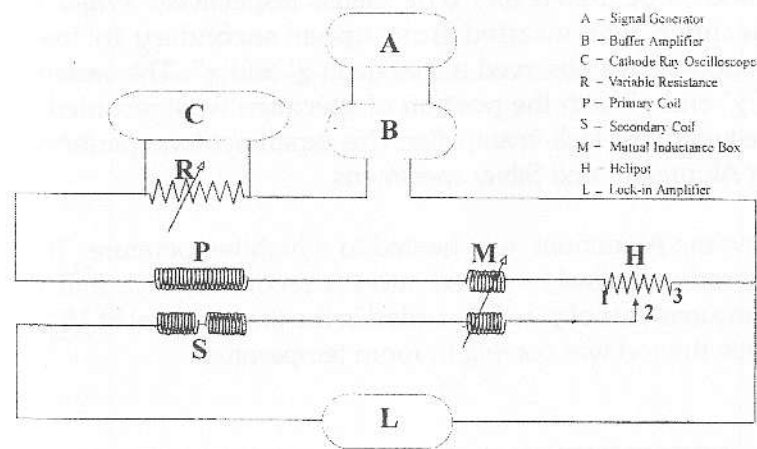


Fig. 2 Diagram for the electronic circuit

R is a variable resistance which is used to alter the a.c. current in the primary coil, while H and M are variable resistor and variable mutual inductance box respectively which are used to balance the bridge. C is a cathode ray oscilloscope which displays the wave form of the a.c. voltage across the resistance R. The excitation voltage of the bridge is produced by an a.c. power supply A and its voltage is amplified by the buffer

amplifier B. The mutual inductance change in the coil system due to the susceptibility of the specimen is recorded by two phase lock-in amplifier L as off balanced voltage.

### Experimental Procedure

For investigation of ac susceptibility on para and diamagnetic material the Aluminum rod of length 20cm and diameter 0.9cm was used as paramagnetic material and the Silver solid of rectangular cross section and of dimensions 1cm×0.15cm×13.5cm was used as diamagnetic material.

Initially the bridge was balanced by varying the frequency of the a.c signal, the helipot and mutual inductance box. The balancing frequency and the ac field in the primary coil were found to be 14kHz and 0.02 Gauss respectively. When the specimen was inserted from upper secondary to lower secondary, we observed a change in  $\chi'$  and  $\chi''$ . The variation of  $\chi'$  and  $\chi''$  with the position of specimen were recorded by the two-phase lock-in amplifier. This experiment was performed for Aluminum and Silver specimens.

Now the Aluminum was heated to a high temperature. Then the rod was slowly inserted into the secondary coils and the measurements of  $\chi'$  was recorded in the time interval of 15 Sec while the rod was cooling to room temperature.

### PRESENTATION OF RESULTS AND DISCUSSION

#### Measurements of $\chi'$ and $\chi''$ on Paramagnetic Sample

The measurements of  $\chi'$  and  $\chi''$  on paramagnetic sample (Aluminum) when the sample moved from the upper secondary to the lower secondary coils are presented in figure 3. These measurements of  $\chi'$  and  $\chi''$  were carried out in various positions of the specimen.

#### $\chi'$ and $\chi''$ on Paramagnetic Sample

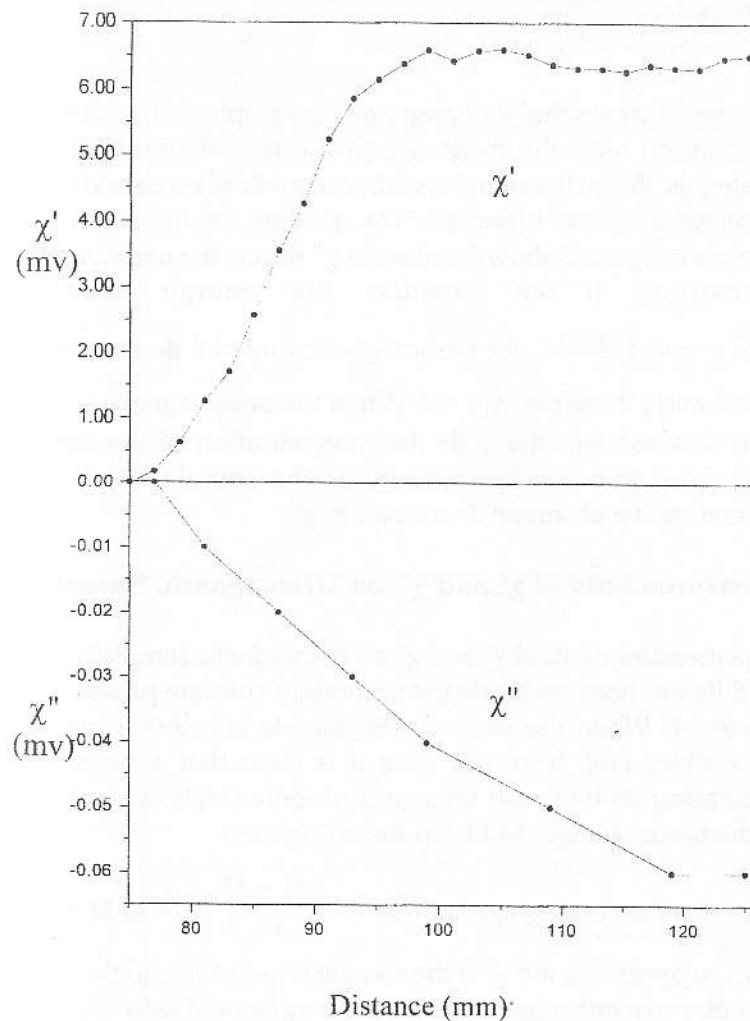


Fig. 3 Measurements of  $\chi'$  and  $\chi''$  at different positions of Aluminum sample (a.c field strength 0.02 Gauss, and a.c frequency 14kHz).

As can be seen from the data in figure.3 a positively increasing off-balanced voltage is observed indicating change in mutual inductance ( $M-M_0$ ) in the coil system.

Therefore magnetic susceptibility of the sample

$$\chi = \frac{M - M_0}{M_0 f} > 0 \text{ as } M - M_0 > 0$$

Our result shows that for paramagnetic sample  $\chi > 0$  which is in agreement with the magnetic properties of paramagnetic materials. When the sample is fully magnetized no considerable changes in  $\chi'$  are observed. The  $\chi''$  data for the Aluminum sample in figure. 3 show decrease in  $\chi''$  during the paramagnetic transition. If we consider the energy equation

$$\Delta G = -\mu_0 \int MdHa, \text{ for paramagnetic material magnetization}$$

$M > 0$  and, therefore  $\Delta G < 0$ . When the paramagnetic sample was inserted into the coils the magnetization of the sample increased and the free energy of the sample decreased. Therefore we observed decreased in  $\chi''$ .

### Measurements of $\chi'$ and $\chi''$ on Diamagnetic Sample

The measurements of  $\chi'$  and  $\chi''$  on diamagnetic sample (Silver) at different position inside the secondary coils are presented in figure. 4. When the diamagnetic sample is inserted into the secondary coil, from our data it is clear that a negatively increasing off balanced voltage is observed indicating mutual inductance change ( $M-M_0$ ) in the coil system.

Therefore according to equation  $\chi = \frac{M - M_0}{M_0 f} < 0$  as  $M - M_0 < 0$

This suggests that the  $\chi'$  of the sample is negative and the Silver is a diamagnetic material which is in agreement with magnetic properties of diamagnetic materials. The  $\chi''$  data for the Silver sample in figure. 4 shows increase in  $\chi''$  during the diamagnetic transition. If we consider the energy equation

$\Delta G = -\mu_0 \int MdHa$ , for diamagnetic material magnetization  $M < 0$  and, therefore  $\Delta G > 0$ . When the diamagnetic sample was inserted the magnetization of the sample decreases and the free energy of the sample increased according to the above equation with  $M < 0$ . Therefore we observed positively increasing energy in figure. 4.

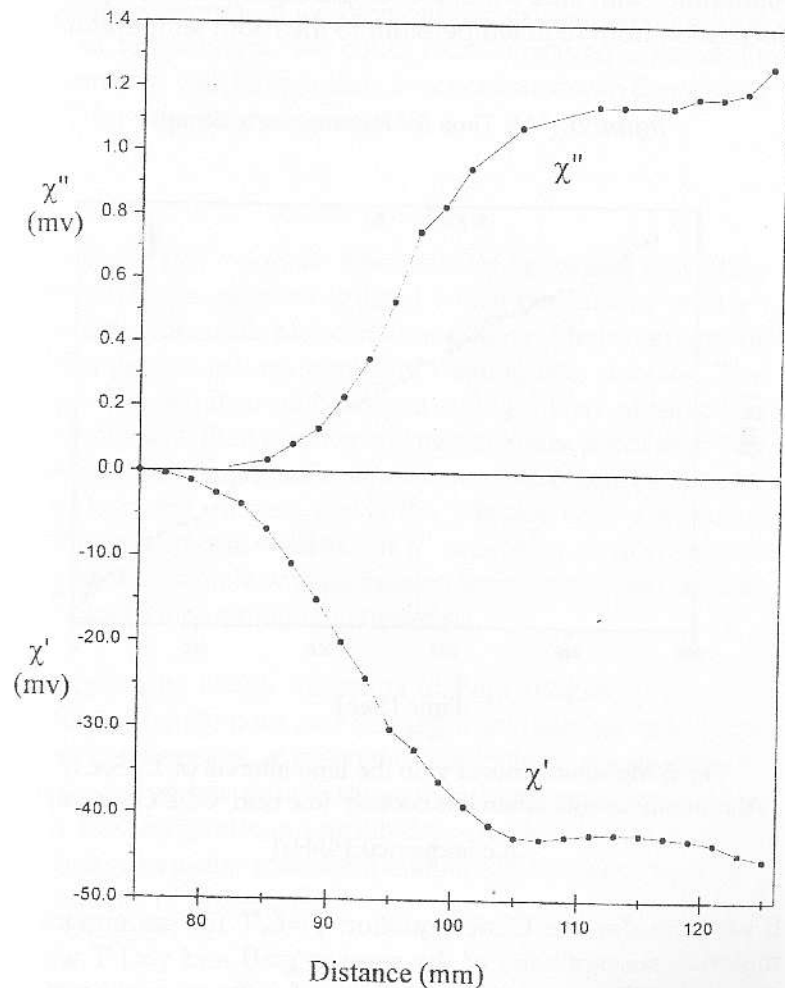


Fig. 4 Measurements of  $\chi'$  and  $\chi''$  at different positions of Silver sample (a.c field strength of 0.02 Gauss and, a.c frequency of 14kHz)

## Measurements of $\chi'$ on Paramagnetic Sample in Different Temperatures

Figure. 5 shows the variation of in-phase component ( $\chi'$ ) of the a.c magnetic susceptibility of the paramagnetic sample, Aluminum, with time when the temperature of the sample was decreasing from high temperature to the room temperature.

figure 5  $\chi'$  Vs Time for Paramagnetic Sample

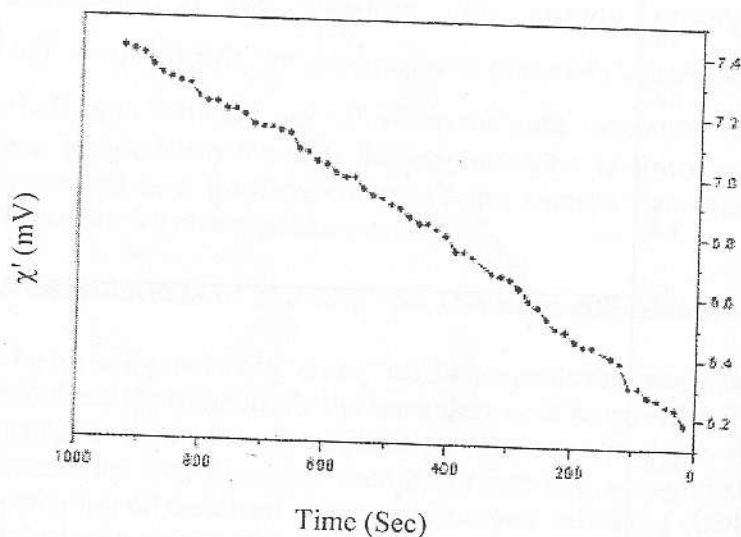


Fig. 5 Measurements of  $\chi'$  in the time interval of 15 Sec of Aluminum sample when it is cooling (a.c field 0.02 Gauss and, a.c frequency 14kHz)

If we consider the Curie equation  $\chi=C/T$  for paramagnetic material, susceptibility of the sample  $\chi>0$  and  $\chi\propto 1/T$  which means that the magnetic susceptibility of a paramagnetic sample is inversely proportional to the temperature of the paramagnetic material.

As can be seen in the data in figure. 5 a positively increasing  $\chi'$  was observed when the sample was cooled. This suggests that the  $\chi'$  of the sample increases with decreasing temperature in accordance with Curie's law for paramagnetic materials. However we have not seen the complete parabolic variation  $\chi=C/T$  in the figure. 5. We have measured  $\chi'$  in a small temperature interval. If we carryout this experiment in a wide range of temperature, we could have observed a parabolic variation in  $\chi'$  with temperature in accordance with Curie's law  $\chi=C/T$  for paramagnetic materials.

## CONCLUSIONS

In this work, AC magnetic susceptibility apparatus consisting Hartshorn bridge was constructed which measures  $\chi'$  and  $\chi''$  of magnetic materials at room temperature. Measurements of  $\chi'$  were carried out on para and diamagnetic samples. The positive and negative off balanced voltages were observed in accordance with their positive and negative susceptibilities. The  $\chi''$  of para and diamagnetic samples were also carried out and energy loss and increase inside the samples were also noted respectively. Measurements of  $\chi'$  was also performed on paramagnetic sample with decreasing temperature and verified Curie's law for paramagnetic materials.

This apparatus which functions at high frequency is more suitable to identify para and diamagnetic materials quickly. In general in researches ac magnetic susceptibility measurements are reported at low temperatures mainly on superconductors which has magnetic susceptibility  $-1$ . But our apparatus functions quiet well at room temperature and measures magnetic susceptibility of order  $10^{-5}$ .

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## REFERENCES

1. Whitemore.S.C, Ryan.S.R (November, 1978) Mutual Inductance Bridge for Low-Temperature Thermometry and Susceptibility Measurements. *Rev.Sci.Instrum* 49(11): 1579-1582
2. Hartshorn.L (1925) *Journal of Scientific Instruments* 5:145.
3. McKim.F.R, Wolf.W.P (February, 1957) An Improved Inductance Method for Measuring Susceptibility of Small Paramagnetic Specimens at Low Temperatures. *Journal of Science Instruments* 34: 64-67.
4. Operating and Service Manual of Lock-in Amplifier Model 5205 &5206, EG & G Brookdeal Electronics. *Princeton Applied Research*: Section I, Section II and Section IV.
5. Najatabadi.B.M (1968) *D.Phil Thesis*, University of Sussex.