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Exact anisotropic sphere with polytropic equation of state

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Abstract. We study static spherically symmetric spacetime to describe compact objects with anisotropic matter distribution. We express the system of Einstein field equations as a new system of differential equations using a coordinate transformation, and then write the system in another form with polytropic equation of state and obtain two classes of exact models. The models satisfy all major physical features expected in a realistic star. For polytropic index $n = 2$, we obtain expressions for mass and density which are comparable with the reported experimental observations.

Keywords. Einstein field equations; anisotropic matter; polytropic equation of state; relativistic star.

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1. Introduction

The description of compact astrophysical objects has been a central issue in relativistic astrophysics for the past decades. Recent experimental observations of radio timing measurements such as a strong Shapiro delay signature in the binary millisecond pulsar J1614-2230 [1] and three post-Keplerian effects in the binary pulsar J1903+0327 [2] suggest that there are compact objects, whose estimated masses $\sim$ (1.5–2) times the solar-mass and radii of $\sim$ (10–15) km give density values that exceed by far the ground state density of atomic nuclei, $\rho_0 \sim 0.16$ nucleons/fm^3 ($\sim 2.5 \times 10^{14}$ g cm^{-3}). These experimental results with many others [3,4] suggest that these pulsar-like stars could be quark stars [5], and are not compatible with the standard neutron star models and rules out some equations of state for superdense matter. Hence there is an increasing challenge for theorists to develop better models to explain the so-called hybrid stars or strange stars that contain quarks in the core either as pure quark matter or as a quark–hadron mixed phase bound by strong interaction, with very different properties from those predicted for hadronic neutron stars [6]. Written [7] was the first to propose that strange quark matter made of up-, down- and strange-quarks is absolutely stable and forms the true ground state of hadronic matter. Strange stars are expected to form during the collapse of the core of a massive star after the supernova explosion. Another possibility is that a rapidly spinning neutron star can accrete sufficient mass to undergo a phase transition to become a strange star.

The description of gravitational collapse and evolution of compact objects under various conditions remain among the important problems of general relativity. The physics of very high density matter is not yet very clear and many of the strange star studies have been performed within the framework of the bag model [8–11]. In the MIT bag model [12], it is assumed that the quark confinement is caused by a universal pressure B on the surface of any region containing quarks and the strange matter equation of state has a simple linear form given by $p = 1/3(\rho - 4B)$ [7], where ρ is the density, p is the isotropic pressure and B is the bag constant. However, the above-stated experimental evidences show that densities within such stars are beyond nuclear matter density, and hence one expects anisotropy to play a major role in theoretically treating such dense objects. Different mechanisms have been identified through the years that create pressure anisotropy in stellar models and make the fluid imperfect [13]. The exotic phase transitions during gravitational collapse [14,15], the existence of a solid core or the presence of a type-P superfluid [16], strong electromagnetic fields [17–19], viscosity [20] as well as the slow rotation of a fluid [21] are some of the mechanisms. In such systems the radial pressure is not equal to tangential pressure. There have been extensive theoretical studies on relativistic compact objects with charged [22–24] and uncharged [25,26] anisotropic matter within the framework of MIT bag model. Study of uncharged anisotropic matter by Sharma and Maharaj [25] shows that for particular parameter values the central density and mass are consistent with that obtained by Dey *et al* [4], who described the quark interaction in a strange star by an interquark vector potential originating from gluon exchange and a density-dependent scalar potential which restores chiral symmetry at high density. In addition to the extensive treatment with linear equation of state, some work has been reported with non-linear equation of state by Varela *et al* [27] and Feroze and Siddiqui [28] on anisotropic matter in the presence of electromagnetic field. Moreover, polytropic model with equation of state, $p = k\rho^{\gamma}$ which was previously studied under the Newtonian gravity [29] and then later extended under general relativity [30], is considered to be stiffer than the conventional bag model, but regarded to be valuable because it could help modelling stars composed of realistic matter, such as ideal gas, photon gas, degenerate Fermi gas and in particular quark matter. Interesting suggestions that quark matter could be in solid form [31], infers that if quarks are clustered in such strange stars where quarks are coupled strongly, the state of cold quark matter might be approximated phenomenologically by polytropic equations of state, which is regarded as an extension to the quark star models with linear equation of state.

Substantial analytical difficulties are associated when polytropic treatment is done for self-gravitating, static, isotropic fluid spheres when pressure explicitly depends on matter density, which invariably leads to non-integrable equations [32]. However, our treatment of anisotropic fluids with polytropic equation of state gets some flexibility in solving the Einstein field equations with uncharged matter in static spherically symmetric spacetime. In §2, we express the system of Einstein field equations as a new system of differential equations using a coordinate transformation, and then write the system in another form with polytropic equation of state which is easier to analyse. Two classes of new exact solutions to the Einstein system are found in §3 in terms of simple elementary functions.

In §4, we impose the physical conditions that must satisfy a physically reasonable model and confirms that the exact solutions found are physically admissible. Some concluding remarks are given in §5.

2. The field equations

We assume that the spacetime manifold is static and spherically symmetric. This assumption is consistent with models used for studying physical behaviours in relativistic astrophysical objects such as dense stars. Consequently, the interior of a spherically symmetric static star is described by the line element

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

in Schwarzschild coordinates $(x^a) = (t, r, \theta, \phi)$. We take the energy–momentum tensor for an anisotropic neutral imperfect fluid sphere to be of the form

$$T_{ij} = \mathrm{diag}(-\rho, p_{\mathrm{r}}, p_{\mathrm{t}}, p_{\mathrm{t}}), \tag{2}$$

where ρ is the energy density, p_{r} is the radial pressure and p_{t} is the tangential pressure. These quantities are measured relative to the co-moving fluid 4-velocity $u^i = e^{-\nu}\delta^i_0$. For the line element (1) and matter distribution (2) the Einstein field equations can be expressed as

$$\frac{1}{r^2}\left[r(1 - e^{-2\lambda})\right]' = \rho, \tag{3}$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_{\mathrm{r}}, \tag{4}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right) = p_{\mathrm{t}}, \tag{5}$$

where primes denote differentiation with respect to r. In the field equations (3)–(5), we are using units where the coupling constant $(8\pi G/c^4) = 1$ and the speed of light $c = 1$. The system of equations (3)–(5) governs the behaviour of the gravitational field for an anisotropic imperfect fluid.

The mass contained within a radius r of the sphere is defined as

$$m(r) = \frac{1}{2}\int_0^r \omega^2\rho(\omega)d\omega. \tag{6}$$

A different but equivalent form of the field equations can be found if we introduce the transformation

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)}, \quad A^2y^2(x) = e^{2\nu(r)}. \tag{7}$$

In (7), the quantities A and C are arbitrary constants. This transformation was first suggested by Durgapal and Bannerji [33]. Under the transformation (7), the system (3)–(5) becomes

$$\frac{1 - Z}{x} - 2\dot{Z} = \frac{\rho}{C}, \tag{8}$$

$$4Z\frac{\dot{y}}{y} + \frac{Z - 1}{x} = \frac{p_{\mathrm{r}}}{C}, \tag{9}$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_{\mathrm{t}}}{C}, \tag{10}$$

where dots denote differentiation with respect to the variable x. The mass function (6) becomes

$$m(x) = \frac{1}{4C^{3/2}} \int_0^x \sqrt{w}\rho(w)\mathrm{d}w \tag{11}$$

in terms of the new variables in (7).

For a physically realistic relativistic star we expect that the matter distribution should satisfy a barotropic equation of state $p_\mathrm{r} = p_\mathrm{r}(\rho)$: in this paper we assume the polytropic equation of state

$$p_\mathrm{r} = k\rho^{1+(1/n)}, \tag{12}$$

where k is a real constant and n is the polytropic index. Then it is possible to write system (8)–(10) in the simpler form

$$\frac{\rho}{C} = \frac{1-Z}{x} - 2\dot{Z}, \tag{13}$$

$$p_\mathrm{r} = k\rho^{1+(1/n)}, \tag{14}$$

$$p_\mathrm{t} = p_\mathrm{r} + \Delta, \tag{15}$$

$$\frac{\Delta}{C} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1 + 2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x}, \tag{16}$$

$$\frac{\dot{y}}{y} = \frac{kC^{1/n}}{4Z}\left[\frac{1-Z}{x} - 2\dot{Z}\right]^{1+(1/n)} + \frac{1-Z}{4xZ}, \tag{17}$$

where the quantity $\Delta = p_\mathrm{t} - p_\mathrm{r}$ is the measure of anisotropy in this model. This system of equations governs the behaviour of the gravitational field for an imperfect fluid source.

3. Exact models

In the system (13)–(17), there are six independent variables (ρ, p_r, p_t, Δ, y, Z) and only five independent equations. This suggests that it is possible to specify one of the quantities involved in the integration process. The resultant system will remain highly nonlinear but it may be possible to generate exact solutions. Equation (17) is the master equation in the integration process. In this treatment we specify the gravitational potential Z so that it is possible to integrate (17). The explicit solution of the Einstein system (13)–(17) then follows. We make a particular choice

$$Z = (1-ax)^2, \tag{18}$$

where a is a real constant. The gravitational potential Z is regular at the origin and well behaved in the stellar interior for a wide range of values for the parameters a. Similar form of gravitational potential were previously used to study a charged perfect fluid source [34]. Therefore, the forms chosen in (18) is physically reasonable. By substituting (18) into (17) we obtain

$$\frac{\dot{y}}{y} = \frac{a(2-ax)}{4(1-ax)^2} + \frac{a^{1+(1/n)}C^{1/n}k(6-5ax)^{1+(1/n)}}{4(1-ax)^2}. \tag{19}$$

In principle, eq. (19) can be integrated if the values of n is specified. We shall consider the following two cases of physical interest.

3.1 *The case* $n = 1$

When $n = 1$, eq. (19) becomes

$$\frac{\dot{y}}{y} = \frac{a(2-ax)}{4(1-ax)^2} + \frac{a^2Ck(6-5ax)^2}{4(1-ax)^2}. \tag{20}$$

On integrating (20) we obtain

$$y = d_1(1-ax)^{-(1+10aCk)/4}\exp\left[\frac{1+aCk\big(1-25(1-ax)^2\big)}{4(1-ax)}\right], \tag{21}$$

where d_1 is the constant of integration. Hence an exact model for the system (13)–(17) is as follows:

$$e^{2\lambda} = \frac{1}{(1-ax)^2}, \tag{22}$$

$$e^{2\nu} = A^2d_1^2(1-ax)^{-(1+10aCk)/2}\exp\left[\frac{1+aCk\big(1-25(1-ax)^2\big)}{2(1-ax)}\right], \tag{23}$$

$$\rho = aC(6-5ax), \tag{24}$$

$$p_{\mathrm{r}} = k\rho^2, \tag{25}$$

$$p_{\mathrm{t}} = p_{\mathrm{r}} + \Delta, \tag{26}$$

$$\Delta = \frac{a^2Cx(6-ax)}{4(1-ax)^2} \times \left\{2 - a\left[x - Ck\left(8 + a[Ck(6-5ax)^3 + 2x(2-5ax)]\right)\right]\right\}. \tag{27}$$

The solutions (22)–(27) are given in simple elementary function so that it may be used to model an anisotropic star with quadratic equation of state.

3.2 *The case* $n = 2$

When $n = 2$, eq. (19) becomes

$$\frac{\dot{y}}{y} = \frac{a(2-ax)}{4(1-ax)^2} + \frac{\sqrt{C}ka^{3/2}(6-5ax)^{3/2}}{4(1-ax)^2}. \tag{28}$$

On integrating (28) we obtain

$$y = d_2\frac{1}{(1-ax)^{1/4}}\left[\frac{\sqrt{6-5ax}+1}{\sqrt{6-5ax}-1}\right]^{15k\sqrt{aC}/8} \times \exp\left[\frac{1-k\sqrt{aC(6-5ax)}(9-10ax)}{4(1-ax)}\right], \tag{29}$$

where d_2 is the constant of integration. Hence an exact model for the system (13)–(17) is as follows:

$$e^{2\lambda} = \frac{1}{(1-ax)^2}, \tag{30}$$

$$e^{2\nu} = A^2 d_2^2 \frac{1}{(1-ax)^{1/2}} \left[\frac{\sqrt{6-5ax}+1}{\sqrt{6-5ax}-1}\right]^{15k\sqrt{aC}/4} \times \exp\left[\frac{1-k\sqrt{aC(6-5ax)}(9-10ax)}{2(1-ax)}\right], \tag{31}$$

$$\rho = aC(6-5ax), \tag{32}$$

$$p_\mathrm{r} = k\rho^{3/2}, \tag{33}$$

$$p_\mathrm{t} = p_\mathrm{r} + \Delta, \tag{34}$$

$$\Delta = \frac{a^2Cx}{4(1-ax)^2}\big[12+5a^2x^2+8a(27Ck^2-2x)+2\sqrt{aC}k\sqrt{6-5ax}(9-8ax) -5a^2Ck^2x(108-90ax+25a^2x^2)\big]. \tag{35}$$

The solutions (30)–(35) also are given in simple elementary function so that it may be used to model a polytropic star.

For both cases the mass function takes the form

$$m(x) = \frac{ax^{3/2}(2-ax)}{2\sqrt{C}}. \tag{36}$$

4. Physical analysis

In this section, we show that the models generated satisfy the physical properties that should be satisfied by a realistic star [35]. The physical properties are:

(i) regularity of the gravitational potentials at the origin;
(ii) positive definiteness of the energy density and the radial pressure at the origin;
(iii) vanishing of the pressure at some finite radius;
(iv) monotonic decrease of the energy density and the radial pressure with increasing radius.
(v) the interior metric match smoothly with the Schwarzschild exterior metric:

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

across the boundary $r = R$, where M is the total mass of the sphere.

4.1 *Physical analysis for the case $n = 1$*

(i) In this model $e^{2\nu(0)} = A^2 d_1^2 \exp\left[\frac{1}{2} - 12aCk\right]$, $e^{2\lambda(0)} = 1$ and $(e^{2\nu(r)})'_{r=0} = (e^{2\lambda(r)})'_{r=0} = 0$. This shows that the gravitational potentials are regular at the origin.

(ii) Since $\rho(0) = 6aC$ and $p_{\rm r}(0) = 36a^2C^2k$, the energy density and radial pressure are positive at the origin if $a > 0$.

(iii) At the boundary of the star $r = R$, the condition $p_{\rm r}(r = R) = 0$ implies $R = \sqrt{6/5aC}$ which is finite for $a > 0$.

(iv) Since $\mathrm{d}\rho/\mathrm{d}r = -10a^2C^2r < 0$ for all $0 < r < R$ and $\mathrm{d}p_{\rm r}/\mathrm{d}r = -20a^3C^3kr(6 - 5aCr^2) < 0$ for all $0 < r < R$, the energy density ρ and the radial pressure decrease monotonically from the centre to the boundary of the star $r = R$.

(v) From (36) we get

$$m(r) = \frac{1}{2}aCr^3(2 - aCr^2).$$

Therefore, the total mass of the star

$$M = m(R) = \frac{12\sqrt{6}}{25\sqrt{5aC}}.$$

Matching conditions imply that

$$\left(1 - \frac{2M}{R}\right)^{-1} = \frac{1}{(1 - aCR^2)^2}, \tag{37}$$

$$\left(1 - \frac{2M}{R}\right) = A^2y^2(CR^2). \tag{38}$$

The condition (37) does not impose any restrictions on the parameters. However, the condition (38) imposes restriction on the parameters a and d as

$$a = \frac{4h - 1}{10Ck} \qquad \text{and} \qquad d_1 = \pm\frac{\mathrm{e}^{5/4}}{5(-5)^hA},$$

where h is an integer.

Thus this model satisfies the physical requirements of a realistic relativistic star in view of general relativity theory.

4.2 *Physical analysis for the case $n = 2$*

(i) In this case $\mathrm{e}^{2\nu(0)} = A^2d_2^2[\frac{1}{5}(7 + 2\sqrt{6})]^{(15/4)k\sqrt{aC}} \exp[\frac{1}{2} - 9\sqrt{\frac{3}{2}}\sqrt{aC}k]$, $\mathrm{e}^{2\lambda(0)} = 1$ and $(\mathrm{e}^{2\nu(r)})' = (\mathrm{e}^{2\lambda(r)})' = 0$ at the origin $r = 0$. This shows that the gravitational potentials are regular at the origin.

(ii) Since $\rho(0) = 6aC$ and $p_{\rm r}(0) = 6\sqrt{6}a^{3/2}C^{3/2}k$, the energy density and radial pressure are positive at the origin if $a > 0$.

(iii) At the boundary of the star $r = R$, the condition $p_{\rm r}(r = R) = 0$ implies $R = \sqrt{6/5aC}$ which is finite for $a > 0$.

(iv) Since $\mathrm{d}\rho/\mathrm{d}r = -10a^2C^2r < 0$ for all $0 < r < R$ and $\mathrm{d}p_{\rm r}/\mathrm{d}r = -15a^{5/2}C^{5/2}kr\sqrt{6 - 5aCr^2} < 0$ for all $0 < r < R$, the energy density ρ and the radial pressure decrease monotonically from the centre to the boundary of the star $r = R$.

(v) It is noted that the energy density and the radius of the star in both models are equal, and hence the total mass of the star (M) remains the same:

$$M = m(R) = \frac{12\sqrt{6}}{25\sqrt{5aC}}.$$

In this case also, the condition (37) does not impose any restrictions on the parameters. However the condition (38) imposes restriction on the parameters a and d as

$$a = \frac{4(4h-1)^2}{225Ck^2} \qquad \text{and} \qquad d_2 = \pm\frac{(-1)^h e^{5/4}}{5\sqrt[4]{5}A}.$$

where h is an integer.

It is noted that, generally, for polytropic models, the polytropic index $n = 1.5$–3 mark the bounds of the most general range of values seen in real stars. A star could be unstable due to strong gravity and hence high pressure ($n < 3$) is necessary for gravitational stability [36]. The lower limit $n = 1.5$ may represent low-mass white dwarfs ($< 1M_\odot$) and main-sequence stars ($< 0.5M_\odot$) [37]. Hence, a more detailed analytical discussion for an intermediate value $n = 2$ would be appropriate.

Therefore, consider the square of sound speed which is

$$\frac{\mathrm{d}p_\mathrm{r}}{\mathrm{d}\rho} = \frac{3k}{2}\sqrt{aC(6-5ax)} \le \frac{3k}{2}\sqrt{6aC} = \frac{\sqrt{6}}{5}|1-4h|$$

in the interior of the star. To maintain the usual casuality, i.e., as the speed of sound being less than the speed of light throughout the interior of the star, we must impose the condition

$$\frac{\mathrm{d}p_\mathrm{r}}{\mathrm{d}\rho} \le \frac{\sqrt{6}}{5}|1-4h| < 1,$$

which gives the constraint on the integer $h = 0$.

We now show that this solution can be used to describe realistic compact objects. In this model, the parameter a has the dimension of length^{-2}. For simplicity, we introduce the transformation

$$\tilde{a} = aS^2,$$

where S is a parameter which has the dimension of length. Under this transformation the energy density becomes

$$\rho = \frac{\tilde{a}}{S^2}(6 - 5\tilde{a}\tilde{x}), \tag{39}$$

where we have set $C = 1$ and $\tilde{x} = r^2/S^2$. Then the mass contained within a radius R has the form

$$M = \frac{12\sqrt{6}}{25\sqrt{5}}\frac{S}{\sqrt{\tilde{a}}}. \tag{40}$$

It is now possible to calculate the central density and mass for particular parameter values from (39) and (40). For example, if we set $\tilde{a} = 0.024$, $S = 1$ km and $k = 0.860531$, we obtain $R = 7.07$ km, the density at the centre $\rho_0 = 7.7194 \times 10^{15}$g cm^{-3} and the mass $M = 2.2976M_\odot$. If we set $\tilde{a} = 0.0071$, $S = 1$ km and $k = 1.5823$, as another example, we obtain the central density $\rho_0 = 2.28296 \times 10^{15}$g cm^{-3} and the mass $M = 4.22496M_\odot$ for $R = 13$ km.

5. Discussion

We have generated two classes of models with the polytropic equation of state which could describe the behaviour of anisotropic compact sphere in static spherically symmetric spacetime. We express the system of Einstein field equations as a new system of differential equations using a coordinate transformation, and then write the system in another form with polytropic equation of state. The generated models satisfy all the major physical features of a realistic star: regularity at the origin of the gravitational potential; positive definiteness of energy density and the radial pressure at the origin; vanishing of radial pressure at some finite radius; and monotonic decrease of the energy density and the radial pressure with increasing radius. We have also shown that these models match smoothly with the Schwarzschild exterior line element at the boundary which restricts the values of the parameters involved in the model. We note that no restrictions have been imposed on the tangential pressure in this study. We also impose the causality condition for $n = 2$ case and obtain expressions for mass and density, which enable us to compare our results with some experimental results.

Now we shall compare the values obtained for central density and mass as examples in §4.2 for $n = 2$, with some experimental results. For $R = 13$ km, the density at the centre $\rho_0 = 2.28296 \times 10^{15}$ g cm^{-3} and the mass $M = 4.2 M_\odot$. It is noted that such high mass could exist in reality [38]. Experimental observations suggest a strange star model for SAX J1808.4-3658, which is estimated to have a mass of $1.44 M_\odot$ for a radius of 7.07 km [39]. It is observed that the mass $M = 2.2976 M_\odot$ obtained with polytropic index $n = 2$ for a radius of 7.07 km suggests an average density of about 1.6 times of SAX J1808.4-3658. Hence, as the solutions generated by this model satisfy all major properties of a realistic star and also give mass and densities comparable with the experimental observations, it could be useful to study the behaviour of a realistic polytropic star.

References

[1] P B Demorest, T Pennucci, S M Ransom, M S E Roberts and J W T Hessels, *Nature* **467**, 1081 (2011)

[2] P C C Freire, C G Bassa, N Wex, I H Stairs, D J Champion, S M Ransom, P Lazarus, V M Kaspi, J W T Hessels, M Kramer, J M Cordes, J P W Verbiest, P Podsiadlowski, D J Nice, J S Deneva, D R Lorimer, B W Stappers, M A McLaughlin and F Camilo, *Mon. Not. R. Astron. Soc.* **412**, 2763 (2011)

[3] J A Pons, F M Walter, J M Lattimer, M Prakash, R Neuhuser and P An, *Astrophys J.* **564**, 981 (2002)

[4] M Dey, I Bombaci, J Dey, S Ray and B C Samanta, *Phys. Lett.* **B438**, 123 (1998)

[5] R Xu, *Mod. Phys. Lett.* **A23**, 1629 (2008)

[6] A Kurkela, P Romatschke and A Vuorinen, *Phys. Rev.* **D81**, 105021 (2010)

[7] E Written, *Phys. Rev.* **D30**, 272 (1984)

[8] N K Glendenning, Ch Kettner and F Weber, *Phys. Rev. Lett.* **74**, 3519 (1995)

[9] Ch Kettner, F Weber, M K Weigel and N K Glendenning, *Phys. Rev.* **D51**, 1440 (1995)

[10] M K Mak and T Harko, *Int. J. Mod. Phys.* **D13**, 149 (2004)

[11] K Komathiraj and S D Maharaj, *Int. J. Mod. Phys.* **D16**, 1803 (2007)

[12] A Chodos, R L Jaff, K Johnson, C B Thorn and V F Weiskopf, *Phys. Rev.* **D9**, 3471 (1974)

[13] L Herrera and N O Santos, *Phys. Rep.* **286**, 53 (1997)

[14] A I Sokolov, *JETP* **79**, 1137 (1980)
[15] L Herrera and L Nez, *Astrophys. J.* **339**, 339 (1989)
[16] R Kippenhahn and A Weigert, *Stellar structure and evolution* (Springer, Berlin, 1990)
[17] F Weber, *Pulsars as astrophysical observatories for nuclear and particle physics* (Institute of Physics, Bristol, 1999)
[18] A Perez Martínez, H Perez Rojas and H J Mosquera Cuesta, *Eur. Phys. J.* **C29**, 111 (2003)
[19] V V Usov, *Phys. Rev.* **D70**, 067301 (2004)
[20] B V Ivanov, *Int. J. Theor. Phys.* **49**, 1236 (2010)
[21] L Herrera and N O Santos, *Astrophys. J.* **438**, 308 (1995)
[22] S Thirukkanesh and S D Maharaj, *Class. Quantum Grav.* **25**, 235001 (2008)
[23] S D Maharaj and S Thirukkanesh, *Pramana – J. Phys.* **72**, 481 (2009)
[24] M Esculpi and E Alom, *Eur. Phys. J.* **C67**, 521 (2010)
[25] R Sharma and S D Maharaj, *Mon. Not. R. Astron. Soc.* **375**, 1265 (2007)
[26] B C Paul, P K Chattopadhyay, S Karmakar and R Tikekar, *Mod. Phys. Lett.* **A26**, 575 (2011)
[27] V Varela, F Rahaman, S Ray, K Chakraborty and M Kalam, *Phys. Rev.* **D82**, 044052 (2010)
[28] T Feroze and A A Siddiqui, *Gen. Relativ. Gravit.* **43**, 1025 (2011)
[29] S Chandrasekhar, *Introduction to the study of stellar structure* (The University of Chicago Press, Chicago, 1939)
[30] R F Tooper, *Astrophys. J.* **142**, 1541 (1965)
S L Shapiro and S A Teukolsky, *Black holes, white dwarfs, and neutron stars: The physics of compact objects* (Wiley-Interscience, New York, 1983)
[31] R X Xu, *Astrophys. J.* **596**, L59 (2003)
J Horvath, *Mod. Phys. Lett.* **A20**, 2799 (2005)
B J Owen, *Phys. Rev. Lett.* **95**, 211101(4) (2005)
M Mannarlli, K Rajagopal and R Sharma, *Phys. Rev.* **D76**, 074026 (2007)
R X Xu and E W Liang, *Sci. China Ser. G: Phys. Mech. Astron.* **52**, 315 (2009)
[32] B V Ivanov, *Phys. Rev.* **D65**, 104001 (2002)
[33] M C Durgapal and R Bannerji, *Phys. Rev.* **D27**, 328 (1983)
[34] S Maurya and Y Gupta, *Astrophys. Space Sci.* **333**, 149 (2011)
[35] M S R Delgaty and K Lake, *Comput. Phys. Commun.* **115**, 395 (1998)
[36] X Y Lai and R X Xu, *Astropart. Phys.* **31**, 128 (2009)
[37] F A Rasio and S L Shapiro, *Astrophys. J.* **438**, 887 (1995)
[38] M T Beltran, R Cesaroni, C Codella, L Testi, R S Furuya and L Olmi, *Nature* **443**, 427 (2006)
[39] X-D Li, I Bombaci, M Dey, J Dey and E P J van den Heuvel, *Phys. Rev. Lett.* **83**, 3776 (1999)