

# Some new static charged spheres

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## Abstract

We present new exact solutions for the Einstein-Maxwell system in static spherically symmetric interior spacetimes. For a particular form of the gravitational potentials and the electric field intensity, it is possible to integrate the system in closed form. For specific parameter values it is possible to find new exact models for the Einstein-Maxwell system in terms of elementary functions. Our model includes a particular charged solution found previously; this suggests that our generalised solution could be used to describe a relativistic compact sphere. A physical analysis indicates that the solutions describe realistic matter distributions.

*Key words:* Exact solutions; Einstein-Maxwell equations; relativistic astrophysics

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## 1 Introduction

Exact solutions of the Einstein-Maxwell system of field equations, for spherically symmetric gravitational fields in static manifolds, are necessary to describe charged compact spheres in relativistic astrophysics. The solutions to the field equations generated have a number of different applications in relativistic stellar systems. It is for this reason that a number of investigations have been undertaken on the Einstein-Maxwell equations in recent times. A comprehensive review of exact solutions and criteria for physical admissability is provided by Ivanov [1]. A general treatment of nonstatic spherically symmetric solutions to the Einstein-Maxwell system, in the case of vanishing shear was, performed by Wafo Soh and Mahomed [2] using symmetry methods. The

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uncharged case was considered by Wafo Soh and Mahomed [3] who show that all existing solutions arise because of the existence of a Noether point symmetry; the physical relevance of the solutions was investigated by Feroze *et al* [4]. The matching of nonstatic charged perfect fluid spheres to the Reissner-Nordstrom exterior metric was pursued by Mahomed *et al* [5] who highlighted the role of the Bianchi identities in restricting the number of solutions.

In this paper, we seek a new class of solutions to the Einstein-Maxwell system that satisfies the physical criteria. We attempt to perform a systematic series analysis to the coupled Einstein-Maxwell equations by choosing a rational form for one of the gravitational potentials and a particular form for the charged matter distribution. This approach produces a number of difference equations, which we demonstrate can be solved explicitly from first principles. A similar approach was used by Thirukkanesh and Maharaj [6], Maharaj and Thirukkanesh [7] and Komathiraj and Maharaj [8]. They obtained exact solutions by reducing the condition of pressure isotropy to a recurrence relation with real and rational coefficients which could be solved by mathematical induction. In this way new mathematical and physical insights in the Einstein-Maxwell field equations were generated. An advantage of this approach is that we generate new solutions to the Einstein-Maxwell system which contain uncharged solutions found previously: Maharaj and Leach [9], Tikekar [10], Durgapal and Bannerji [11] and John and Maharaj [12], amongst others.

We first express the Einstein-Maxwell system of equations for static spherically symmetric line element as an equivalent system using the Durgapal and Bannerji [11] transformation in Section 2. In Section 3, we choose specific forms for one of the gravitational potentials and the electric field intensity, which reduce the condition of pressure isotropy to a linear second order equation in the remaining gravitational potential. We integrate this generalised condition of isotropy equation using the method of Frobenius in Section 4. In general the solution will be given in terms of special functions. We demonstrate that it is possible to find two category of solutions in terms of elementary functions by placing certain restriction on the parameters. We regain known charged Einstein-Maxwell models from our general class of models in Section 5. In Section 6, we discuss the physical features of the solutions found, plot the matter variables, and show that our models are physically reasonable.

## 2 The field equations

We assume that the spacetime is spherically symmetric and static which is consistent with the study of charged compact objects in relativistic astrophysics. In Schwarzschild coordinates  $(t, r, \theta, \phi)$  the generic form of the line

element is given by

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

The Einstein field equations for the line element (1) can be written as

$$\frac{1}{r^2} [r(1 - e^{-2\lambda})]' = \rho + \frac{1}{2}E^2, \quad (2a)$$

$$-\frac{1}{r^2} (1 - e^{-2\lambda}) + \frac{2\nu'}{r} e^{-2\lambda} = p - \frac{1}{2}E^2, \quad (2b)$$

$$e^{-2\lambda} \left( \nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r} \right) = p + \frac{1}{2}E^2, \quad (2c)$$

$$\sigma = \frac{1}{r^2} e^\lambda (r^2 E)', \quad (2d)$$

for charged perfect fluids. The energy density  $\rho$  and the pressure  $p$  are measured relative to the comoving fluid 4-velocity  $u^a = e^{-\nu} \delta_0^a$  and primes denote differentiation with respect to the radial coordinate  $r$ . The quantities associated with the electric field are  $E$ , the electric field intensity, and  $\sigma$ , the proper charge density. In the system (2a)-(2d), we are using units where the coupling constant  $\frac{8\pi G}{c^4} = 1$  and the speed of light  $c = 1$ . This system of equations determines the behaviour of the gravitational field for a charged perfect fluid source. A different, but equivalent form of the field equations, can be found if we introduce the transformation

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)}, \quad A^2 y^2(x) = e^{2\nu(r)}, \quad (3)$$

where the parameters  $A$  and  $C$  are arbitrary constants. Under the transformation (3) the system (2a)-(2d) has the equivalent form

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{E^2}{2C}, \quad (4a)$$

$$4Z\frac{\dot{y}}{y} + \frac{Z-1}{x} = \frac{p}{C} - \frac{E^2}{2C}, \quad (4b)$$

$$4Zx^2\ddot{y} + 2\dot{Z}x^2\dot{y} + \left( \dot{Z}x - Z + 1 - \frac{E^2x}{C} \right) y = 0, \quad (4c)$$

$$\frac{\sigma^2}{C} = \frac{4Z}{x} (x\dot{E} + E)^2, \quad (4d)$$

where dots denote differentiation with respect to  $x$ . This system of equations governs the behaviour of the gravitational field for a charged perfect fluid source. When  $E = 0$  the Einstein-Maxwell equations (4a)-(4d) reduce to the uncharged Einstein equations for a neutral fluid. Equation (4c) is called the generalised condition of pressure isotropy and is the fundamental equation

that needs to be integrated to demonstrate an exact solution to the Einstein-Maxwell system of equations (4a)-(4d).

### 3 Choosing potentials

Our objective is to find a new class of solutions to the Einstein-Maxwell system by making explicit choices for the gravitational potential  $Z$  and the electric field intensity  $E$ . We make the choice for  $Z$  as

$$Z(x) = \frac{(1 + ax)^2}{1 + bx}, \quad (5)$$

where  $a$  and  $b$  are real constants. Note that the choice (5) ensures that the gravitational potential  $e^{2\lambda}$  is regular and well behaved in the stellar interior for a wide range of values of the parameters  $a$  and  $b$ . In addition, when  $x = 0$  then  $Z = 1$  which ensures that there is no singularity at the stellar centre. A special case of (5) was studied by Komathiraj and Maharaj [13]. The choice (5) does produce charged and uncharged solutions which are necessary for constructing realistic stellar models. On substituting (5) in (4c) we obtain

$$4(1 + ax)^2(1 + bx)\ddot{y} + 2(1 + ax)[b(1 + ax) - 2(b - a)]\dot{y} + \left[ (a - b)^2 - \frac{E^2(1 + bx)^2}{Cx} \right] y = 0, \quad (6)$$

which is a second order differential equation.

The differential equation (6) may be solved if a particular choice of the electric field intensity  $E$  is made. For our purpose we set

$$\frac{E^2}{C} = \frac{\alpha a(b - a)x}{(1 + bx)^2}, \quad (7)$$

where  $\alpha$  is a constant. The electric field intensity specified in (7) vanishes at the centre of the sphere; it is continuous and bounded in the stellar interior for wide range of values of  $x$ . The quantity  $E^2$  has positive values in the interior of star for relevant choices of the constants  $\alpha$ ,  $a$  and  $b$ . Therefore the form given in (7) is physically reasonable to study the behaviour of charged spheres. With the choice (7) we can express (6) in the form

$$4(1 + ax)^2 [b(1 + ax) - (b - a)] \ddot{y} + 2a(1 + ax) [b(1 + ax) - 2(b - a)] \dot{y} + a(b - a)(b - a - \alpha a)y = 0. \quad (8)$$

In (8) we assume that  $a \neq 0$  and  $a \neq b$  so that the electric field intensity is present. When  $\alpha = 0$  there is no charge.

## 4 Solutions

To find the solution of the Einstein-Maxwell system we need to integrate the master equation (8). We consider two cases on the integration process:  $\alpha = \frac{b}{a} - 1$  and  $\alpha \neq \frac{b}{a} - 1$ .

### 4.1 The case $\alpha = \frac{b}{a} - 1$

In this case the differential equation (8) becomes

$$2(1+ax)[b(1+ax) - (b-a)]\ddot{y} + a[b(1+ax) - 2(b-a)]\dot{y} = 0. \quad (9)$$

Equation (9) is easily integrable and the solution can be written as

$$y(x) = c_1 \left( \sqrt{\frac{a(1+bx)}{b-a}} - \arctan \sqrt{\frac{a(1+bx)}{b-a}} \right) + c_2, \quad (10)$$

where  $c_1$  and  $c_2$  are constants of integration. Therefore, the solution of the Einstein-Maxwell system (4a)-(4d) becomes

$$e^{2\lambda} = \frac{1+bx}{(1+ax)^2}, \quad (11a)$$

$$e^{2\nu} = A^2 \left[ c_1 \left( \sqrt{\frac{a(1+bx)}{b-a}} - \arctan \sqrt{\frac{a(1+bx)}{b-a}} \right) + c_2 \right]^2, \quad (11b)$$

$$\frac{\rho}{C} = \frac{(b-2a)(6+bx)}{2(1+bx)^2} - \frac{a^2x(11+6bx)}{2(1+bx)^2}, \quad (11c)$$

$$\begin{aligned} \frac{p}{C} = & \frac{(2a-b)(2+bx)}{2(1+bx)^2} + \frac{a^2x(3+2bx)}{2(1+bx)^2} \\ & + \frac{2ac_1(1+ax)\sqrt{\frac{a(1+bx)}{b-a}}}{c_1(1+bx) \left( \sqrt{\frac{a(1+bx)}{b-a}} - \arctan \sqrt{\frac{a(1+bx)}{b-a}} \right) + c_2}, \end{aligned} \quad (11d)$$

$$\frac{E^2}{C} = \frac{(b-a)^2x}{(1+bx)^2}. \quad (11e)$$

Observe that because of the restrictions  $\alpha = \frac{b}{a} - 1$  and  $b \neq a$  the charged solution (11) does not have an uncharged limit. Therefore this solution models

a sphere that is always charged and cannot attain a neutral state. Note that the solution (11) is expressed in a simple form in terms of elementary functions which facilitates a physical analysis of the matter and gravitational variables.

#### 4.2 The case $\alpha \neq \frac{b}{a} - 1$

With  $\alpha \neq \frac{b}{a} - 1$ , equation (8) is difficult to solve. Consequently we introduce the transformation

$$y = (1 + ax)^d U(1 + ax), \quad (12)$$

where  $U$  is a function of  $(1 + ax)$  and  $d$  is constant. With the help of (12), the differential equation (8) can be written as

$$\begin{aligned} & 4(1 + ax)^2 [b(1 + ax) - (b - a)] \ddot{U} \\ & + 2(1 + ax) [b(4d + 1)(1 + ax) - 2(2d + 1)(b - a)] \dot{U} \\ & + \left[ 2bd(2d - 1)(1 + ax) - (b - a) \left( \frac{b}{a} - 1 - \alpha - 4d^2 \right) \right] U = 0. \end{aligned} \quad (13)$$

Note that there is substantial simplification if we take

$$\frac{b}{a} - 1 - \alpha = 4d^2.$$

Then (13) becomes

$$\begin{aligned} & 2(1 + ax) \left[ (1 + ax) - \frac{(b - a)}{b} \right] \ddot{U} \\ & + \left[ (4d + 1)(1 + ax) - 2(2d + 1) \frac{(b - a)}{b} \right] \dot{U} + d(2d - 1)U = 0, \end{aligned} \quad (14)$$

where  $b \neq 0$ . We observe that the point  $1 + ax = \frac{b-a}{b}$  is a regular singular point of the differential equation (14). Therefore, the solution of the differential equation (14) can be written in the form of an infinite series by the method of Frobenius:

$$U = \sum_{i=0}^{\infty} c_i \left[ (1 + ax) - \frac{(b - a)}{b} \right]^{i+r}, \quad c_0 \neq 0, \quad (15)$$

where  $c_i$  are the coefficients of the series and  $r$  is the constant. To complete the solution we need to find the coefficients  $c_i$  as well as the parameter  $r$  explicitly.

The indicial equation determines the value of  $r$  from

$$c_0 r(2r - 3) = 0.$$

As  $c_0 \neq 0$  we must have  $r = 0$  or  $r = 3/2$ . We can express the structure for the general coefficient  $c_i$  in terms of the leading coefficient  $c_0$  as

$$c_i = \left(\frac{b}{a-b}\right)^i \prod_{p=1}^i \frac{[(p+r-1)(2p+2r+4d-3) + d(2d-1)]}{(p+r)(2p+2r-3)} c_0, \quad (16)$$

where the conventional symbol  $\prod$  denotes multiplication. We can verify the result (16) using mathematical induction. We can now generate two linearly independent solutions to (14) with the help of (15) and (16). For the parameter value  $r = 0$ , we obtain the first solution

$$U_1 = c_0 \left[ 1 + \sum_{i=1}^{\infty} \left(\frac{b}{a-b}\right)^i \prod_{p=1}^i \frac{[(p-1)(2p+4d-3) + d(2d-1)]}{p(2p-3)} \times \left[ (1+ax) - \frac{(b-a)}{b} \right]^i \right].$$

For the parameter value  $r = 3/2$ , we obtain the second solution

$$U_2 = c_0 \left[ (1+ax) - \frac{(b-a)}{b} \right]^{3/2} \left[ 1 + \sum_{i=1}^{\infty} \left(\frac{b}{a-b}\right)^i \prod_{p=1}^i \frac{[(2p+1)(p+2d) + d(2d-1)]}{p(2p+3)} \left[ (1+ax) - \frac{(b-a)}{b} \right]^i \right].$$

Since the functions  $U_1$  and  $U_2$  are linearly independent we have found the general solution to (14). Therefore, the solutions to the differential equation (8) are

$$y_1(x) = c_0(1+ax)^d \left[ 1 + \sum_{i=1}^{\infty} \left(\frac{b}{a-b}\right)^i \times \prod_{p=1}^i \frac{[(p-1)(2p+4d-3) + d(2d-1)]}{p(2p-3)} \left[ (1+ax) - \frac{(b-a)}{b} \right]^i \right] \quad (17)$$

and

$$y_2(x) = c_0(1+ax)^d \left[ (1+ax) - \frac{(b-a)}{b} \right]^{3/2} \left[ 1 + \sum_{i=1}^{\infty} \left(\frac{b}{a-b}\right)^i \times \prod_{p=1}^i \frac{[(2p+1)(p+2d) + d(2d-1)]}{p(2p+3)} \left[ (1+ax) - \frac{(b-a)}{b} \right]^i \right]. \quad (18)$$

Thus the general solution to the differential equation (6), for the choice of the electric field (7), is given by

$$y(x) = A_1 y_1(x) + A_2 y_2(x), \quad (19)$$

where  $A_1$  and  $A_2$  are arbitrary constants and  $d^2 = \left(\frac{b}{a} - 1 - \alpha\right)/4$ . From (19) and (4a)-(4d), the exact solution of the Einstein-Maxwell system becomes

$$e^{2\lambda} = \frac{1 + bx}{(1 + ax)^2}, \quad (20a)$$

$$e^{2\nu} = A^2 y^2, \quad (20b)$$

$$\frac{\rho}{C} = \frac{(3 + bx)(b - 2a)}{(1 + bx)^2} - \frac{a^2 x(5 + 3bx)}{(1 + bx)^2} - \frac{\alpha a(b - a)x}{2(1 + bx)^2}, \quad (20c)$$

$$\frac{p}{C} = 4 \frac{(1 + ax)^2}{(1 + bx)} \frac{\dot{y}}{y} + \frac{a(2 + ax) - b}{(1 + bx)} + \frac{\alpha a(b - a)x}{2(1 + bx)^2}, \quad (20d)$$

$$\frac{E^2}{C} = \frac{\alpha a(b - a)x}{(1 + bx)^2}. \quad (20e)$$

We believe that this is a new solution to the Einstein-Maxwell system. In general the models in (20) cannot be expressed in terms of elementary functions as the series in (17) and (18) do not terminate. Consequently the solution will be given in terms of special functions. Terminating series are possible for particular values of  $a$  and  $b$  as we show in the next section.

## 5 Elementary functions

It is possible to generate exact solutions in terms of elementary functions from the series in (19). This is possible for specific values of the parameters  $a$ ,  $b$  and  $\alpha$  so that the series (17) and (18) terminate. Consequently two categories of solutions are obtainable in terms of elementary functions by placing restrictions on the quantity  $\frac{b}{a} - 1 - \alpha$ . We can express the first category of solution, in terms of the variable  $r$ , as



$$\begin{aligned}
y_1(x) &= A_1 \frac{1}{(1+ax)^n} \times \\
&\sum_{i=0}^n (-1)^{i-1} \left(\frac{b}{b-a}\right)^i \frac{(2i-1)}{(2i)!(2n-2i+1)!} \left[ (1+ax) - \frac{(b-a)}{b} \right]^i \\
&+ A_2 \frac{1}{(1+ax)^n} \left[ (1+ax) - \frac{(b-a)}{b} \right]^{3/2} \times \\
&\sum_{i=0}^{n-1} \left(\frac{b}{a-b}\right)^i \frac{(i+1)}{(2i+3)!(2n-2i-2)!} \left[ (1+ax) - \frac{(b-a)}{b} \right]^i, \quad (21)
\end{aligned}$$

where  $\frac{b}{a} - 1 - \alpha = 4n^2$  relates the constants  $a, b, \alpha$  and  $n$ . The second category of solution, in terms of the variable  $r$ , is given by

$$\begin{aligned}
y_2(x) &= A_1 \frac{1}{(1+ax)^{n-1/2}} \times \\
&\sum_{i=0}^n (-1)^{i-1} \left(\frac{b}{b-a}\right)^i \frac{(2i-1)}{(2i)!(2n-2i)!} \left[ (1+ax) - \frac{(b-a)}{b} \right]^i \\
&+ A_2 \frac{1}{(1+ax)^{n-1/2}} \left[ (1+ax) - \frac{(b-a)}{b} \right]^{3/2} \times \\
&\sum_{i=0}^{n-2} \left(\frac{b}{a-b}\right)^i \frac{(i+1)}{(2i+3)!(2n-2i-3)!} \left[ (1+ax) - \frac{(b-a)}{b} \right]^i, \quad (22)
\end{aligned}$$

where  $\frac{b}{a} - 1 - \alpha = 4n(n-1) + 1$  relates the constants  $a, b, \alpha$  and  $n$ . Thus we have extracted two classes of solutions (21) and (22) to the Einstein-Maxwell system in terms of elementary functions from the infinite series solution (19). This class of solution can be expressed as combinations of polynomials and algebraic functions. The simple form of (21) and (22) helps in the study of the physical features of the model.

From our general classes of solutions (21) and (22), it is possible to generate particular solutions found for charged stars previously. If we take  $b = 1$  and  $K = \frac{1-a}{a}$  then it is easy to verify that the equation (21) becomes

$$\begin{aligned}
y_1(x) &= D_1 \left[ \frac{K}{K+1+x} \right]^n \sum_{i=0}^n (-1)^{i-1} \frac{(2i-1)}{(2i)!(2n-2i+1)!} \left[ \frac{1+x}{K} \right]^i \\
&+ D_2 \left[ \frac{K}{K+1+x} \right]^n \left[ \frac{1+x}{K} \right]^{3/2} \times \\
&\sum_{i=0}^{n-1} (-1)^i \frac{(i+1)}{(2i+3)!(2n-2i-2)!} \left[ \frac{1+x}{K} \right]^i, \quad (23)
\end{aligned}$$

where  $K - \alpha = 4n^2$ ,  $D_1 = \frac{A_1}{(1-a)^n}$  and  $D_2 = \frac{A_2}{(1-a)^{n-3/2}}$ . Also, equation (22)

becomes

$$\begin{aligned}
y_2(x) = & D_1 \left[ \frac{K}{K+1+x} \right]^{n-1/2} \sum_{i=0}^n (-1)^{i-1} \frac{(2i-1)}{(2i)!(2n-2i)!} \left[ \frac{1+x}{K} \right]^i \\
& + D_2 \left[ \frac{K}{K+1+x} \right]^{n-1/2} \left[ \frac{1+x}{K} \right]^{3/2} \times \\
& \sum_{i=0}^{n-2} (-1)^i \frac{(i+1)}{(2i+3)!(2n-2i-3)!} \left[ \frac{1+x}{K} \right]^i, \tag{24}
\end{aligned}$$

where  $K - \alpha = 4n(n-1) + 1$ ,  $D_1 = \frac{A_1}{(1-a)^{n-1/2}}$  and  $D_2 = \frac{A_2}{(1-a)^{n-2}}$ . Thus we have regained the Komathiraj and Maharaj [13] charged model; our solutions allow for a wider range of models for charged relativistic spheres. We illustrate this feature with an example involving a specific value for  $n$ . For example, suppose that  $n = 1$  then  $b = (5 + \alpha)a$  and we get

$$y = \frac{a_1(7 + \alpha + 3(5 + \alpha)ax) + a_2(1 + (5 + \alpha)ax)^{3/2}}{1 + ax} \tag{25}$$

from (21) where  $a_1$  and  $a_2$  are new arbitrary constants. From (25) and (4a)-(4d) the solution to the Einstein-Maxwell system becomes

$$e^{2\lambda} = \frac{1 + (5 + \alpha)ax}{(1 + ax)^2}, \tag{26a}$$

$$e^{2\nu} = A^2 \left[ \frac{a_1(7 + \alpha + 3(5 + \alpha)ax) + a_2(1 + (5 + \alpha)ax)^{3/2}}{1 + ax} \right]^2, \tag{26b}$$

$$\begin{aligned}
\frac{\rho}{C} = & \frac{a(3 + \alpha - ax)}{1 + (5 + \alpha)ax} + \frac{2a(1 + ax)[3 + \alpha - (5 + \alpha)ax]}{[1 + (5 + \alpha)ax]^2} \\
& - \frac{\alpha a^2(4 + \alpha)x}{2[1 + (5 + \alpha)ax]^2}, \tag{26c}
\end{aligned}$$

$$\begin{aligned}
\frac{p}{C} = & \frac{2a(1 + ax)}{[1 + (5 + \alpha)ax]} \times \\
& \frac{[4a_1(4 + \alpha) + a_2(1 + (5 + \alpha)ax)^{1/2}(13 + 3\alpha + (5 + \alpha)ax)]}{[a_1(7 + \alpha + 3(5 + \alpha)ax) + a_2(1 + (5 + \alpha)ax)^{3/2}]} \\
& - \frac{a(3 + \alpha - ax)}{1 + (5 + \alpha)ax} + \frac{\alpha a^2(4 + \alpha)x}{2[1 + (5 + \alpha)ax]^2}, \tag{26d}
\end{aligned}$$

$$\frac{E^2}{C} = \frac{\alpha a^2(4 + \alpha)x}{[1 + (5 + \alpha)ax]^2}. \tag{26e}$$

Note that the solution of the form (26) cannot be regained from Komathiraj and Maharaj [13] charged models except for the value of  $a = \frac{1}{(5+\alpha)}$ . This indi-

cates that our model is the generalisation of Komathiraj and Maharaj charged models with more general behaviour in the gravitational and electromagnetic fields.

## 6 Physical analysis

In this section, we briefly consider the physical features of the models generated in this paper. For the pressure to vanish at the boundary  $r = R$  in the solution (20) we require  $p(R) = 0$  which gives the condition

$$4(1 + aCR^2)^2 \left[ \frac{\dot{y}}{y} \right]_{r=R} + a(2 + aCR^2) - b + \frac{\alpha a(b - a)CR^2}{2(1 + bCR^2)} = 0, \quad (27)$$

where  $y$  is given by (17)-(19). This will constrain the values of  $a, b$  and  $\alpha$ . The solution of the Einstein-Maxwell system for  $r > R$  is given by the Reissner-Nordstrom metric as

$$ds^2 = - \left( 1 - \frac{2m}{r} + \frac{q^2}{r^2} \right) dt^2 + \left( 1 - \frac{2m}{r} + \frac{q^2}{r^2} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (28)$$

where  $m$  and  $q$  are the total mass and the charge of the star. To match the potentials in (20) to (28) generates the relationships between the constants  $A_1, A_2, a, b$  and  $R$  as follows

$$\left( 1 - \frac{2m}{R} + \frac{q^2}{R^2} \right) = A^2 [A_1 y_1(R) + A_2 y_2(R)]^2, \quad (29a)$$

$$\left( 1 - \frac{2m}{R} + \frac{q^2}{R^2} \right)^{-1} = \frac{1 + bCR^2}{(1 + aCR^2)^2}. \quad (29b)$$

The matching conditions (27) and (29) place restrictions on the metric coefficients; however there are sufficient free parameters to satisfy the necessary conditions that arise for the model under study. Since these conditions are satisfied by the constants in the solution a relativistic star of radius  $R$  is realisable.

From (20a) and (20b) we easily observe that the gravitational potentials  $e^{2\lambda}$  and  $e^{2\nu}$  are continuous and well behaved for wide range of the parameters  $a$  and  $b$ . From (20c), the variable  $x$  can be expressed solely in terms of the energy density  $\rho$  as

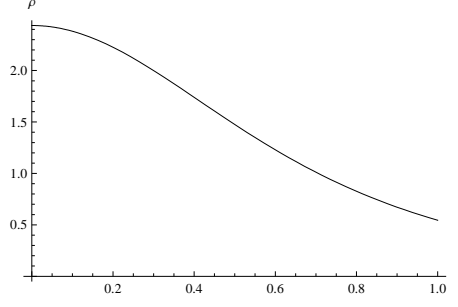


Fig. 1. Energy density.

$$x = \frac{1}{2b(3a^2C + b\rho)} \left[ b^2C - 2b\rho - 5a^2C - 2abC \pm \sqrt{(a-b)C} \times \sqrt{[-27a^2bC + a^3C(25 + 6b\alpha) - b^2(bC + 8\rho) + ab(3bC + 8\rho + 2b\alpha\rho)]} \right]$$

Hence, from (20d) the isotropic pressure  $p$  can be written as a function of energy density  $\rho$  only. Therefore the solutions generated in this paper satisfy the barotropic equation of state  $p = p(\rho)$ . Many of the solutions found previously do not satisfy this desirable feature. We illustrate the graphical behaviour of matter variables in the stellar interior for the particular solution (26). We assume that  $a_1 = -4.897$ ,  $a_2 = C = 1$  and  $a = \alpha = 1/4$  for simplicity, and we consider the interval  $0 \leq r \leq 1$ . To generate the plots for  $\rho, p, E^2, dp/d\rho$  and  $p$  vs  $\rho$ , we utilised the software package Mathematica. The behaviour of the energy density is plotted in Fig. 1. It is positive and monotonically decreasing towards the boundary of the stellar object. In Fig. 2, we have plotted the behaviour of matter pressure  $p$ , which is regular, monotonically decreasing and becomes zero at the vacuum boundary of the stellar object. In Fig. 3, we describe the behaviour of the electric field intensity. It is well behaved and a continuous function. In Fig. 4, we have plotted the speed of sound  $dp/d\rho$ . We observe that  $0 \leq dp/d\rho \leq 1$  throughout the interior of the stellar object. Therefore the speed of the sound is less than the speed of the light and causality is maintained. In Fig. 5, we have plotted the pressure  $p$  versus the density  $\rho$  and we find that this approximates a linear function. This behaviour is to be expected as the gradients of  $p$  and  $\rho$  have similar profiles in the stellar interior. Thus we have demonstrated that the particular solution satisfies the requirements for a physically reasonable stellar interior in the context of general relativity.

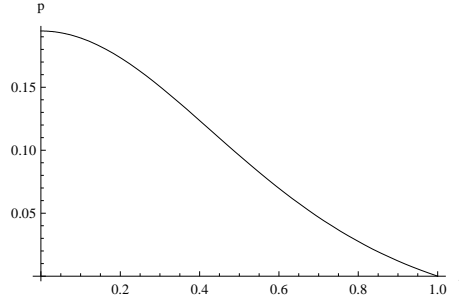


Fig. 2. Matter pressure.

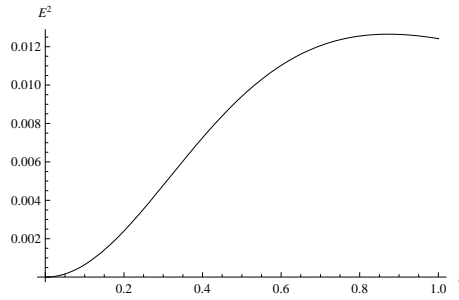


Fig. 3. Electric field intensity.

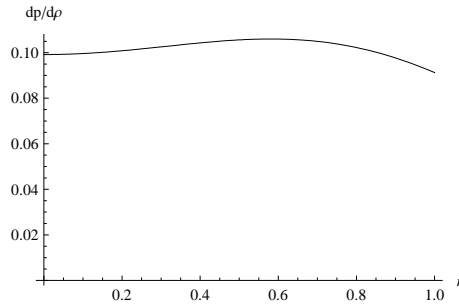


Fig. 4. Speed of sound  $\frac{dp}{d\rho}$ .

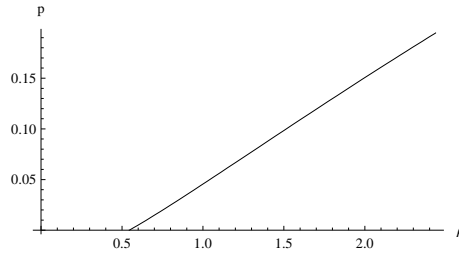


Fig. 5. Pressure vs Density

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