EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2014/2015

FIRST SEMESTER (Aug./Sep., 2016)

PM 101 - FOUNDATION OF MATHEMATICS

Proper & Repeat

Answer all questions | Time : Three hours

---

1. (a) Using truth tables, show that $p \vee q \equiv \sim p \longrightarrow q$.

   Hence rewrite the following compound statements using only the connectives $\wedge$ and $\sim$:

   i. $(\sim p \ \vee \ q) \longrightarrow (r \vee \sim q)$;

   ii. $(p \longrightarrow r) \ \longleftrightarrow \ (q \longrightarrow r)$.

   (b) Prove the following equivalences using the laws of logic:

   i. $\sim [p \vee (\sim p \wedge q)] \ \equiv \ \sim p \wedge \sim q$;

   ii. $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \ \equiv \ \sim p$,

   where p and q are statements.

   (c) Using the valid argument forms, show that the following argument is valid:

   "If you send me an e-mail message, then I will finish writing the program. If you do not send me an e-mail message, then I will go to sleep early. If I go to sleep early, then I will wake up feeling refreshed. Therefore, If I do not finish writing the program, then I will wake up feeling refreshed."

2. (a) For any sets A and B, prove that $A \triangle B = (A \cup B) \setminus (A \cap B)$.

   Hence, show that:

   i. $A \cup B = (A \triangle B) \cup (A \cap B)$,

   ii. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

(b) For any sets A, B and C, prove that:

i. $A \times (B \cap C) = (A \times B) \cap (A \times C)$,

ii. $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

3. (a) Let R be a relation defined on $\mathbb{C} \setminus \{0\}$ by $z_1 R z_2 \Longleftrightarrow z_1\bar{z}_1(z_2 + \bar{z}_2) = z_2$ where $\mathbb{C}$ denotes the set of all complex numbers. Prove that R is an relation.

If a is a non-zero real number, show that $R-$class of a is a circle wit $\left(\frac{1}{2}a, 0\right)$ and radius $\frac{1}{2}a$.

(b) i. Prove that every partially ordered set has at most one last element.

ii. Show that last element of every partially ordered set is a maximal

Is the converse true? Justify your answer.

4. (a) Define the following terms:

i. *injective mapping*,

ii. *surjective mapping*,

iii. *inverse mapping*.

(b) The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \begin{cases} 1 - x, & \text{if } x \geq 0; \\ x^2, & \text{if } x < 0; \end{cases} \quad \text{and } g(x) = \begin{cases} x, & \text{if } x \geq \\ x - 1, & \text{if } x < \end{cases}$$

Find the formula for $f \circ g$

Show that $f \circ g$ is a bijection and give a formula for $(f \circ g)^{-1}$.

5. Let $f : X \to Y$ be a mapping and A, B subsets of X. Prove the following:

(a) $f(A \cup B) = f(A) \cup f(B)$;

(b) $f(A \cap B) \subseteq f(A) \cap f(B)$;

(c) Is $f(A) \cap f(B) \subseteq f(A \cap B)$ in general? Justify your answer;

(d) f is surjective iff $Y \setminus f(A) \subseteq f(X \setminus A)$.

6. (a) Let a, b and c be integers. If $a \mid b$ and $a \mid c$ then prove that $a \mid (bx +$ integers x and y.

Hence, show that the greatest common divisor of $a + 2b$ and $2a + b$ is 1 b are relatively prime..

(b) Using the Euclidean algorithm find integers x and y satisfying

$$gcd(341, 527) = 341x + 527y.$$

(c) Find all the integer solutions of the Diophantine equation $21x + 7y =$