EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE(2014/2015)

FIRST SEMESTER (Aug./Sept., 2016)

AM 106 - TENSOR CALCULUS

Proper & Repeat

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ver all question — **Time: One hour**

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1) Define the Covariant tensor A_{pq} and the Contravariant tensor A^{pq}.

2) Express the relationship between the following associated tensors:

   i. A^{ijk} and A_{pqr};

   ii. $A_j^{\ k}{}_l$ and A^{qkr}.

3) Let A_p, B_r^{qs} be an arbitrary tensors. Show that if $A^p B_r^{qs} C(p,q,r,s,)$ is an invariant then $C(p,q,r,s)$ is a mixed tensor. What is its rank?

4) Find g and g^{jk} corresponding to the line element

$$ds^2 = 2(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 + 4dx^1dx^2 - 2dx^1dx^3.$$

2. (a) Define the following:

   i. Christofell's symbols of the first and second kind;

   ii. Geodesic.

   Explain the terms "Covariant derivative" as applied to the tensor A^p_r.

   (b) With the usual notations, prove the following:

   i. $[p\,q, r] = g_{rs}\Gamma^s_{p\,q}$;

   ii. $\dfrac{\partial g^{p\,q}}{\partial x^m} + g^{p\,n}\Gamma^q_{m\,n} + g^{q\,n}\Gamma^p_{m\,n} = 0.$

   Deduce that the covariant derivatives of $g_{j\,k}$, $g^{j\,k}$ and δ^j_k are zero.

   (c) Show that the non-vanishing Christofel's symbols of the second kind in cylin coordinate (ρ, ϕ, z) are given by

$$\Gamma^1_{22} = -\rho, \quad \Gamma^2_{21} = \frac{1}{\rho}, \quad \Gamma^2_{12} = \frac{1}{\rho},$$

   where $x^1 = \rho, \quad x^2 = \phi, \quad x^3 = z.$