# EASTERN UNIVERSITY, SRI LANKA

## DEPARTMENT OF MATHEMATICS

## FIRST EXAMINATION IN SCIENCE - 2013/2014

## FIRST SEMESTER (Sep./Oct., 2015)

## PM 101 - FOUNDATION OF MATHEMATICS

## (PROPER/REPEAT)



**Answer all questions** **Time : Three hours**

1. (a) Let $p$ and $q$ be two statements such that $p \rightarrow \sim q$ is false. Find the truth value of each of the following statements:

   i. $p \wedge (q \rightarrow \sim p)$;

   ii. $q \wedge (p \vee \sim q)$.

   (b) Prove the following equivalences using the laws of logic:

   i. $(p \wedge q) \vee \sim p \equiv \sim p \vee q$;

   ii. $[p \vee (q \wedge r)] \vee \sim [(\sim q \wedge \sim r) \vee r] \equiv p \vee q$,

   here $p, q$ and $r$ are statements.

   (c) Using the valid argument forms, deduce the conclusion $t$ from the premises given below:

$$p \vee q$$

$$q \rightarrow r$$

$$p \wedge s \rightarrow t$$

$$\sim r$$

$$\sim q \rightarrow u \wedge s,$$

   here $p, q, r, s, t$ and $u$ are statements.

2. (a) Simplify the following expressions using the laws of sets:

   i. $[(A \cup \Phi) \cap (B \cup A') \cap (A \cup B \cup X)]'$;

   ii. $[A \cap (B \cup C)] \cap [(B' \cup C') \cap C]'$,

   here $A, B$ and $C$ are subsets of a universal set $X$.

   (b) For any sets $A$ and $B$, prove that $A \bigtriangleup B = (A \cup B) \setminus (A \cap B)$.

   Hence show that:

   i. $A \bigtriangleup B$ and $A \cap B$ are disjoint;

   ii. $A \cup B = (A \bigtriangleup B) \cup (A \cap B)$.

   (c) Prove the following:

   i. $(A \times C) \cup (B \times C) = (A \cup B) \times C$;

   ii. $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$.

3. (a) Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \neq 0, y \neq 0\}$ and define a relation $R$ on
   $(x_1, y_1)R(x_2, y_2)$ if and only if $x_1y_1(x_2^2 - y_2^2) = x_2y_2(x_1^2 - y_1^2)$.

   i. Show that $R$ is an equivalence relation;

   ii. If $(a, b)$ is a fixed element of $S$, show that
   $(x, y)R(a, b)$ if and only if $\frac{a}{b} = \frac{x}{y}$ or $\frac{a}{b} = -\frac{y}{x}$.

   (b) If $\lambda$ and $\mu$ are equivalence relations on a set $A$, prove that:

   i. $\lambda \cap \mu$ is an equivalence relation on $A$;

   ii. $\lambda \cup \mu$ need not be an equivalence relation on $A$.

   (c) Show that the last element of every partially ordered set is a maximal element
   Is the converse true? Justify your answer.

4. (a) Define what is meant by the following terms:

   i. *injective mapping*,

   ii. *surjective mapping*,

   iii. *inverse mapping*.

   (b) The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \begin{cases} 4x + 1, & \text{if } x \geq 0; \\ x, & \text{if } x < 0; \end{cases} \quad \text{and } g(x) = \begin{cases} 3x, & \text{if } x \geq 0; \\ x + 3, & \text{if } x < 0. \end{cases}$$

   Show that $g \circ f$ is a bijection and give a formula for $(g \circ f)^{-1}$.

5. Let $f : X \to Y$ be a mapping and $A$ and $B$ be any subsets of $X$. Prove the following:

(a) $f(A \cap B) \subseteq f(A) \cap f(B)$;

(b) In general, $f(A \cap B) \neq f(A) \cap f(B)$.

(c) $f$ is injective if and only if $f(A \cap B) = f(A) \cap f(B)$;

(d) $f$ is surjective if and only if $Y \setminus f(A) \subseteq f(X \setminus A)$.

6. (a) State the *division algorithm.*

Show that the square of any odd integer is of the form $8k + 1$, where $k$ is an integer.

(b) Define the *greatestcommondivisor*, $gcd(a, b)$, of two integers $a$ and $b$ not both zero.

Using the Euclidean algorithm find the $gcd(341, 527)$.

Hence express the $gcd(341, 527)$ as a linear combination of 341 and 527.

(c) 1000 glasses are packed in two types of boxes. There are 172 boxes in the first type and 20 in the second type. If each type contains a fixed number of glasses, find the number of glasses in each type.