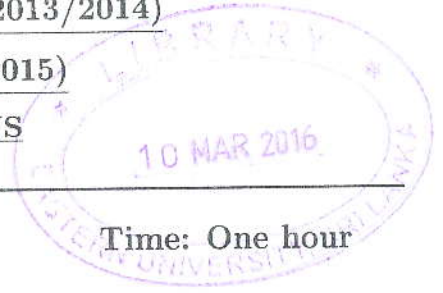




EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
FIRST EXAMINATION IN SCIENCE(2013/2014)
FIRST SEMESTER (Sept./Oct., 2015)
AM 106 - TENSOR CALCULUS



Answer all question

Time: One hour

- (a) Define the following terms:
- i. Covariant tensor;
 - ii. Contravariant tensor.
- (b) Let A_{rst}^{pq} be a tensor.
- i. Choose $p = t$ and show that A_{rsp}^{pq} , where the summation convention employed, is a tensor. What is its rank?
 - ii. Choose $p = t, q = s$ and show similarly A_{rqp}^{pq} is a tensor. What is its rank?
- (c) The covariant components of a tensor in rectangular co-ordinate system are $x^2 - y, 2x - z^2, xyz$. Find its covariant components in cylindrical co-ordinate system.
- (d) Let A_p, B_r^{qs} be an arbitrary tensors. Show that if $A^p B_r^{qs} C(p, q, r, s)$ is an invariant then $C(p, q, r, s)$ is a mixed tensor of rank four.

2. (a) Define the following:

- i. Christoffel's symbols of the first and second kind;
- ii. Geodesic;
- iii. Covariant derivative of a tensor A_p .

(b) With the usual notations, prove the following:

- i. $[pq, r] = g_{rs}\Gamma_{pq}^s$;
- ii. $[pm, q] + [qm, p] = \frac{\partial g_{pq}}{\partial x^m}$;
- iii. $\frac{\partial g^{pq}}{\partial x^m} + g^{pn}\Gamma_{mn}^q + g^{qn}\Gamma_{mn}^p = 0$.

(c) Prove that the covariant derivatives of g_{jk} , g^{jk} and δ_k^j are zero.