EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2011/2012

FIRST SEMESTER (Jan./Feb, 2014)

MT 101 - FOUNDATION OF MATHEMATICS

(RE-REPEAT)

Answer all questions — Time : Three hours

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1. (a) Let p and q be two statements such that $p \rightarrow \sim q$ is false. Find the truth value of each of the following statements:

   i. $p \wedge (q \rightarrow \sim p)$;

   ii. $q \wedge (p \vee \sim q)$.

   (b) Prove the following equivalences using the laws of algebra of logic:

   i. $(p \wedge q) \vee \sim p \equiv \sim p \vee q$;

   ii. $[p \vee (q \wedge r)] \vee \sim [(\sim q \wedge \sim r) \vee r] \equiv p \vee q$,

   where p, q and r are statemtnts.

   (c) Using the valid argument forms, deduce the conclusion t from the premises given below:

$$p \vee q$$
$$q \rightarrow r$$
$$p \wedge s \rightarrow t$$
$$\sim r$$
$$\sim q \rightarrow u \wedge s,$$

   where p, q, r, s, t and u are statements.

2. (a) For any sets A and B, prove that $A \triangle B = (A \cup B) \setminus (A \cap B)$.

Hence show that:

i. $A \triangle B$ and $A \cap B$ are disjoint,

ii. $A \cup B = (A \triangle B) \cup (A \cap B)$.

(b) For any sets A, B and C, prove that:

i. $A \times (B \cap C) = (A \times B) \cap (A \times C)$,

ii. $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

3. (a) Let ρ be a relation defined on $\mathbb{R}$ by $x \rho y \Leftrightarrow x^2 - y^2 = 2(y - x)$, where $\mathbb{R}$ denotes set of all real numbers.

i. Prove that ρ is an equivalence relation.

ii. Determine the ρ–class of 1.

(b) Let R be an equivalence relation on a set A. Prove the following:

i. $[a] \neq \Phi \quad \forall a \in A$,

ii. $aRb \Leftrightarrow [a] = [b]$,

iii. either $[a] = [b]$ or $[a] \cap [b] = \Phi \quad \forall a \in A$.

4. (a) Define the following terms:

i. *injective mapping*,

ii. *surjective mapping*,

iii. *inverse mapping*.

(b) The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \begin{cases} 4x + 1, & \text{if } x \geq 0; \\ x, & \text{if } x < 0; \end{cases} \quad \text{and } g(x) = \begin{cases} 3x, & \text{if } x \geq 0; \\ x + 3, & \text{if } x < 0. \end{cases}$$

Show that $g \circ f$ is a bijection and give a formula for $(g \circ f)^{-1}$.

5. (a) Let $f : X \to Y$ be a mapping. Prove that f is surjective iff $Y \setminus f(A) \subseteq f(X \setminus A$ all subset A of X.

(b) i. Prove that every partially ordered set has at most one last element.

ii. Show that last element of every partially ordered set is a maximal element

Is the converse true? Justify your answer.

6. (a) State *division algorithm.*

Show that the square of any odd integer is of the form $8k+1$, where k is an integer.

(b) Using the Euclidean algorithm find integers x and y satisfying

$$gcd(341, 527) = 341x + 527y.$$

(c) A customer bought 12 pieces of fruits, apples and oranges, for Rs. 132. If an apple costs Rs. 3 more than an orange, and more apples than oranges were purchased, how many pieces of each kind were bought.