EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE -2009/2010

SECOND SEMESTER- (April / May, 2012)

MT 107 – THEORY OF SERIES

(Repeat)

Answer for **all** questions. Time: Two hours.

01. (a) Define the terms convergence and divergence of an infinite series of real numbers $\sum_{n=1}^{\infty} a_n$.

(b) If series $\sum_{n=1}^{\infty} a_n$ converges, show that $a_n \to 0$ as $n \to \infty$.

(c) Check the convergence of the following series:

(i) $2 + \frac{3}{2^3} + \frac{4}{3^3} + \frac{5}{4^3} + \dots\dots\dots$;

(ii) $\sum_{n=1}^{\infty} \frac{1}{4n^2 + 2n - 6}$.

02. (a) State and prove the limit comparison test. Use this test to show that the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}$$

is convergent.

(b) Write down the ratio test and use it to test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2 + 2n + 1}{3^n + 2}$.

(c) Use root test to check the convergence of the series $\sum_{n=1}^{\infty} e^{-n^3 - n^{n^2}}$.

(P. T. O.)

03. (a) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \sin nx$ is absolutely convergent.

(b) Show that the alternative series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent, if

(i) $(a_n)_{n=1}^{\infty}$ is a monotonically decreasing sequence; and

(ii) $\lim_{n \to \infty} a_n = 0$.

(c) Define the convergence of a power series $\sum_{n=1}^{\infty} a_n (x-a)^n$. Find the radius of convergence and interval of convergence of the following power series:

(i) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{5^n \sqrt{n+1}}$;

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^n n(x+3)^n}{4^n}$.

04. (a) Let the series $\sum_{n=1}^{\infty} M_n$ be a convergent series of non-negative real numbers. If $\sum_{n=1}^{\infty} Z_n$ is a series of complex numbers such that $Z_n = x_n + i y_n$, $n \in \mathbb{N}$ and $\left| Z_n \right| \leq M_n$ for all $n \in \mathbb{N}$, then show that $\sum_{n=1}^{\infty} Z_n$ is convergent.

Hence, check whether the series $\sum_{n=1}^{\infty} \frac{(n+i)(1+ni)}{n^2}$ converges or diverges.

(b) Show that the complex-valued function $f(Z)$, which has derivatives of all order, can be expressed in a Taylor series

$$f(Z) = \sum_{n=0}^{\infty} \frac{f^n(Z_0)}{n!} (Z - Z_0)^n$$

where Z is a complex number and $f^n(Z_0)$ is the n[th] derivative of $f(Z)$ at $Z = Z_0$.

Hence expand $\ln\left(\frac{1+Z}{1-Z}\right)$ in a Taylor series about $Z_0 = 0$.

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