06 NOV 2015

EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE(2012/2013)

FIRST SEMESTER (Feb./Mar., 2015)

AM 106 - TENSOR CALCULUS

Answer all question   Time: One hour

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1. (a) Define what is meant by the terms *symmetric tensor, skew symmetric tensor* and *invariant*.

    If $ds^2 = g_{jk}\, dx^j\, dx^k$ is an invariant, then prove that g_{jk} is a symmetric covariant tensor of rank two.

   (b) Given that the covariant components of a tensor in rectangular coordinates system are $yz, 3, 2x + y$. Find its covariant components in cylindrical coordinates system.

   (c) Let A^p and B_r^{qs} be an arbitrary tensors. Show that if $A^p B_r^{qs} C(p, q, r, s,)$ is an invariant then $C(p, q, r, s)$ is a mixed tensor of rank four.

2. (a) Define what it means by the following:

    i. *Christoffel's symbols of first and second kind*;

    ii. *Geodesics*.

    Explain the term covariant derivative as applied to a tensor of type A_r^p.

   (b) With usual notations, prove the following;

    i. $[pq, r] = [qp, r]$;

    ii. $\dfrac{\partial g^{rk}}{\partial x^m} = -g^{ir}\, \Gamma_{im}^k - g^{jk}\, \Gamma_{jm}^r$.

    Hence deduce that the covariant derivative of metric tensors g^{rk} and g_{rk} are zero.

   (c) Determine the Christoffel's symbols of the second kind in spherical coordinates (r, θ, ϕ), and find the corresponding geodesic equations.