EASTERN UNIVERSITY, SRI LANKA

SECOND YEAR EXAMINATION IN SCIENCE - 2012/2013

FIRST SEMESTER(February/March, 2016)

PM 203 - EIGENSPACES AND QUADRATIC FORMS

(PROPER & REPEAT)

Answer all Questions | Time: Two hours

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1. (a) Define what is meant by the terms *eigenvalue* and *eigenvector* of a linear transformation $T : V \longrightarrow V$, where V is a vector space.

   Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & 11 \end{pmatrix}.$$

   (b) i. Prove that the eigenvectors that corresponding to distinct eigenvalues of a linear transformation are linearly independent.

   ii. Prove that all the eigenvalues in the set of all complex numbers, $\mathbb{C}$ of a real orthogonal matrix have modulus 1.

   iii. Let A and B be n-square matrices. Show that AB and BA have the same eigenvalues.

   (c) Orthogonally diagonalize the matrix

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}.$$

2. Define what is meant by the term *minimum polynomial* of a square matr

(a) State and prove the *Cayley-Hamilton* theorem.

Find the minimum polynomial of the square matrix

$$\begin{pmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}.$$

(b) Prove that if $m(t)$ is a minimum polynomial of an $n \times n$ matrix A

is the characteristic polynomial of A, then $\psi_A(t)$ divides $[m(t)]^n$.

(c) Let $M = \begin{pmatrix} A & O_1 \\ O_2 & B \end{pmatrix}$, where A, B are square matrices and O_1, O_2

zero matrices of respective orders. Show that the minimum polyn

of M is the least common multiple of the minimum polynomials $g($

of A and B, respectively.

3. (a) Find an orthogonal transformation which reduces the following qua

to a diagonal form

$$2x_1^2 - 2x_1x_3 + 2x_2^2 - 2x_2x_3 + 3x_3^2 = 16.$$

(b) Simultaneously diagonalize the following pair of quadratic forms

$$\phi_1 = x_1^2 - x_2^2 - 2x_3^2 - 2x_1x_2 + 4x_2x_3,$$

$$\phi_2 = x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3.$$

4 (a) Define what is meant by an *inner product* on a vector space.

Let $x = (x_1, x_2, ..., x_n)$, $y = (y_1, y_2, ..., y_n) \in \mathbb{R}^n$, where $x_i, y_i \in \mathbb{R}$, $i = 1, 2, ..., n$.

Let the inner product $< .,. >$ be defined on $\mathbb{R}^n$ as

$$< x, y >= xy^T = \sum_{i=1}^{n} x_i y_i.$$

Show that $(\mathbb{R}^n, < .,. >)$ is an inner product space.

(b) Prove that for any vectors x, y in an inner product space, $| < x, y > | \leq \|x\| \|y\|$.

(c) State the *Gram Schmidt* process.

Find the orthonormal set for span of M in $\mathbb{R}^4$, where

$$M = \{(1, 1, 1, 1)^T, (1, 1, 2, 4)^T, (1, 2, -4, -3)^T\}.$$