EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SECOND EXAMINATION IN SCIENCE - 2012/2013
FIRST SEMESTER (Feb./Mar., 2016)
MT 207 - NUMERICAL ANALYSIS
(REPEAT)

nswer all Questions — Time: Two hours

1. (a) Define what is meant by:

   i. *absolute error*;

   ii. *relative error* .

   (b) i. Show that the polynomial nesting technique can be used to evaluate

   $$f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99.$$

   ii. Use three - digit rounding arithmetic and the formula given in the statement of part (i) to evaluate $f(1.53)$. Evaluate the absolute error and relative error.

   iii. Repeat the calculation in part (ii) using the nesting form of $f(x)$ that was found in part (i) . Compare the approximations with part (ii).

2. (a) Let $x = \phi(x)$ be, the rearrangement of the equation $f(x) = 0$ and d

iteration,

$$x_{n+1} = \phi(x_n), \qquad n = 0, 1, ...$$

with the initial value x_0. If $\phi'(x)$ exists and is continuous such that $|\phi'(x)|$

for all x, then show that the sequence (x_n) generated by the above

converges to the unique root α of the equation $f(x) = 0$.

The function $g(x) = x^3 - x^2 - 4x + 5$ has three fixed points. One of them

i. Find the other two fixed points.

ii. Does fixed point iteration of g converge for x_0 near 1? Explain.

(b) i. Obtain Newton Raphson method to compute the root of the above

in an interval $[a, b]$. Then use it to approximate the solution to x-

with an error of at most 10^{-4}.

ii. Define the order and the asymptotic error constant of the iteration

to compute the non linear equation

$$f(x) = 0.$$

Hence show that the asymptotic error constant of the Newton

method is $\frac{1}{2}\frac{f''(\alpha)}{f'(\alpha)}$.

3. (a) Suppose that $x_0, x_1, . . ., x_n$ are distinct numbers in the interval [

$f \in C^{n+1}[a, b]$. Obtain a unique polynomial $P_n(x)$ of degree at most

property

$$f(x_k) = P_n(x_k) \qquad \text{for each } k = 0, 1, 2, . . . , n$$

and show that

$$f(x) - P_n(x) = (x - x_0)(x - x_1). . .(x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!},$$

where $\xi \in [a, b]$.

(b) i. Use Lagrange's method to find the interpolating polynomial for the data:

i	0	1	2	3
x_i	1	2	3	4
$\ln x_i$	0	0.693	1.099	1.386

ii. Approximate $\ln(2.718)$ using the polynomial obtained in part (i).

iii. Find an upper bound on the error for the Lagrange interpolating polynomial on the interval $[1, 4]$.

4. (a) Use the Jacobi method to approximate the solution of the following system of linear equations.

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 + 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3 \end{aligned}$$

Continue the iterations until two successive approximations are identical when rounded to three significant digits.

(b) With the usual notations, the Simpson's rule is given by

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = \frac{h}{3}(f_{i-1} + 4f_i + f_{i+1}) - \frac{1}{90}h^5 f^{(iv)}(\xi_i), \text{ where } \xi_i \in [x_{i-1}, x_{i+1}].$$

Obtain the composite Simpson's rule, and show that the composite error is less than or equal to

$$\frac{1}{180}h^4(b-a)\left|f^{(iv)}(\xi)\right|, \text{ where } \left|f^{(iv)}(\xi)\right| = \max_{a\leq x\leq b}\left|f^{(iv)}(x)\right|.$$

A missile is launched from a ground station. The acceleration during its 80 seconds of flight, as recorded, is given in the following table:

$t(s)$	0	10	20	30	40	50	60	70	80
$a(ms^{-2})$	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

compute the velocity of the missile when $t = 80$, using Simpson's $\frac{1}{3}$ rule.