

EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE – 2013/2014

SECOND SEMESTER (OCTOBER 2016)

PH 207 ELECTRICITY AND MAGNETISM II



Time: 01 hour.

Answer ALL Questions

You may find the following information useful:

Electron charge  $e = 1.6 \times 10^{-19} \text{ C}$

Electron mass  $m = 9.1 \times 10^{-31} \text{ kg}$

Permittivity of free space  $\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$

Divergence theorem

$$\int_S \vec{A} \cdot \overline{da} = \int_V \vec{\nabla} \cdot \vec{A} \, d\tau$$

Stokes theorem

$$\oint_C \vec{A} \cdot \overline{dl} = \int_S (\vec{\nabla} \times \vec{A}) \cdot \overline{da}$$

Q1.

Show that in a dielectric material, the bound surface charge density  $\sigma_b$  and volume charge density  $\rho_b$  are expressed by  $\sigma_b = \vec{P} \cdot \vec{n}$  and  $\rho_b = -\vec{\nabla} \cdot \vec{P}$ , where  $\vec{P}$  is the polarization vector.

If the total charge density of a dielectric material is given by  $\rho_{\text{total}} = \rho_f + \rho_b$  (where  $\rho_f$  is the free volume charge density), then prove the following relations:

$$\text{Displacement vector } \vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad \text{and}$$

$$\text{Total free charge } Q_f = \oint_S \vec{D} \cdot \vec{d}\vec{a}.$$

A spherical conducting shell has inner radius  $R_1$  and outer radius  $R_2$ . The region between the spherical surfaces is filled with a medium having a permittivity,

$$\epsilon(r) = \frac{\epsilon_0}{1 + \lambda r}$$

where  $\lambda$  and  $\epsilon_0$  are constants and  $r$  is the radial coordinate. A charge  $Q$  is placed on the surface of the inner shell. When the outer surface is grounded,

- i. The electric field  $\vec{E}$  in the region  $R_1 < r < R_2$
- ii. The displacement vector  $\vec{D}$  in the region  $R_1 < r < R_2$
- iii. The potential difference between the spherical surfaces
- iv. The polarization vector  $\vec{P}$  in the region  $R_1 < r < R_2$
- v. The capacitance of the capacitor
- vi. Bound volume charge density
- vii. Bound surface charge density at  $r = R_1$  and  $r = R_2$

The electric field  $\vec{E}$  in a conducting medium with conductivity  $\sigma$ , permeability  $\mu$  and permittivity  $\epsilon$  satisfies the wave equation

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}.$$

Consider the solution of the above equation a travelling wave in  $x$  direction as  $\vec{E} = \vec{E}_0 e^{i(\omega t - kx)}$ . Show that the wave vector  $k$  and angular frequency  $\omega$  satisfy the dispersion relation:

$$\omega^2 \mu \epsilon = i\omega \mu \sigma + k^2.$$

When the electric wave is travelling in an ionized gas with  $\epsilon = \epsilon_0$  and  $\mu = \mu_0$  it is equivalent to free space,

Show that the dispersion relation becomes

$$\frac{k}{\omega} = \sqrt{\left(1 - \frac{\omega_g^2}{\omega^2}\right) \epsilon_0 \mu_0}$$

where,  $\omega_g$  is the frequency of ionized gas.

Find the frequency of the ionized gas.

Determine the refractive index of the medium, when the electron concentration is  $2.5 \times 10^{10} \text{ m}^{-3}$  and the frequency of the wave is 3 MHz.